Lesson 2 Second Condition

Lesson 2 THE SECOND CONDITION OF EQUILIBRIUM (Rotational)

NOTES

Even though an object does not change its velocity (translational equilibrium), it can still rotate. Consider a seesaw at the playground. The seesaw has no translational motion, but it can still have rotational motion. If an object is in equilibrium, it has translational and rotational equilibrium. If an object is in static equilibrium, it has no translational or rotational motion.

$W = F_{\parallel} d$
[J]

The first condition of equilibrium is $\sum \vec{F} = 0$.

TORQUE

$\tau = F_{\perp} d$
[N·m]

always find component of F that is perpendicular to distance along beam.

The second condition of equilibrium states that in order to have no rotation, there can be no torque. For an object to have rotational equilibrium, the sum of the torques must be zero.

Think about removing a tight nut from a bolt. It will be easier to remove this nut by using a long wrench. And of course, it depends on how you apply a force to this wrench—it is best if you exert the force at right angles to the wrench.

τ

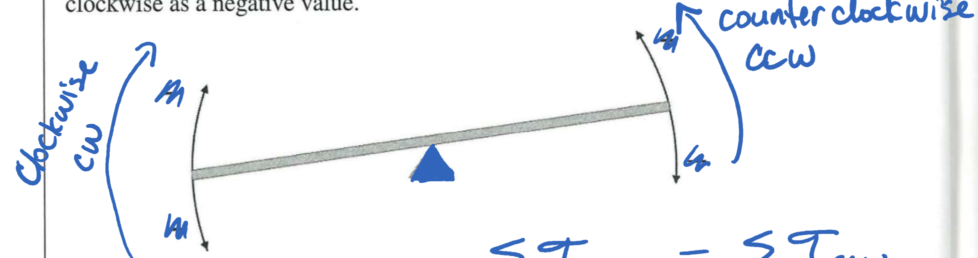
The factors affecting the rotation of the nut are the perpendicular component of the applied force and the length of the wrench (radius of rotation). That is the distance from axis of rotation is perpendicular to the line along which the force acts.
The symbol for magnitude of torque is the Greek letter $\boxed{\text{tau: } \tau.}$

$\tau = rF\cancel{\sin}\theta$

Formula: $\tau = rF\cancel{\sin}\theta$

You will note that torque is the product of the radius of rotation, the magnitude of the force, and the direction of the force.
The unit of torque is: N·m

Torque is a vector quantity; that is, it has direction. For convenience, consider a torque that will cause an object to rotate counterclockwise as a positive value, and consider a torque that will cause an object to rotate clockwise as a negative value.



$\Sigma \tau_{cw} = \Sigma \tau_{ccw}$

if this is true, then there is no rotation (equilibrium)

NOTES

The second condition of equilibrium is expressed as
$\Sigma\tau = 0$

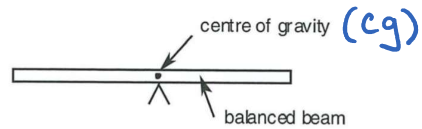
Three additional concepts need to be considered in order to apply the second condition of equilibrium to problems:

- centre of gravity
- arbitrary position of the point of rotation
- lever arm

The second condition of equilibrium is $\Sigma\tau = 0$

CENTRE OF GRAVITY

In torque problems, assume that the weight of an object acts at one point. This one point is called the centre of gravity.
In cases involving a beam, the beam's centre of gravity is the point where it would achieve balance.



In the problems involving torque in this workbook, a uniform beam is one where the centre of gravity is at the centre of the beam.

$\tau = F_{\perp} d$
always measured from the fulcrum

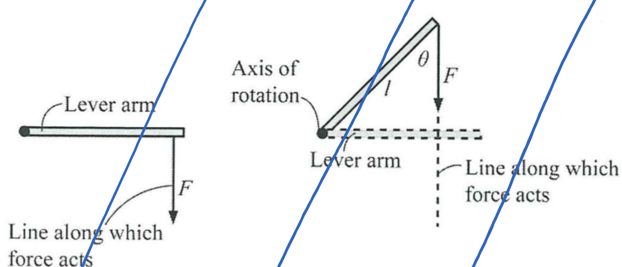
ARBITRARY POSITION OF THE POINT OF ROTATION

In problems on torque, the point of rotation is usually placed at a hinge, a fulcrum etc. But in some problems, the point of rotation may need to be assigned arbitrarily.

- fulcrum
- pivot point

LEVER ARM

The lever arm is the perpendicular distance from the axis of rotation to the line along which the force acts.



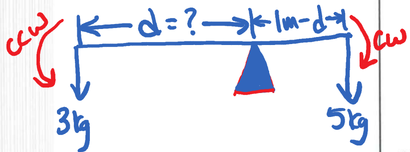
- We calculate the component of F instead

Note: In the figure, the lever arm = $r \sin\theta$
In the equation for torque, $\tau = rF \sin\theta$, $r \sin\theta$ = lever arm.

2 NOTES

Simpler Example:

zero mass meter-stick with masses hanging from ends.



$$\Sigma \tau_{ccw} = \Sigma \tau_{cw}$$

$$F_{\perp 3} d = F_{\perp 5}(1-d)$$

$$(3kg)(9.81\tfrac{N}{kg})d = (5kg)(9.81\tfrac{N}{kg})(1-d)$$

$$3d = 5 - 5d$$

$$8d = 5$$

$$d = \frac{5}{8} = 0.625\,m$$

Example 1

A torque of 24.0 N·m is needed to tighten a nut. If a person applies a force of 100 N, what is the minimum length of wrench required?

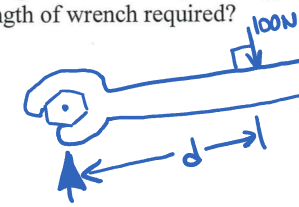
Solution

$\tau = rF\cancel{\sin\theta}$

$r = \dfrac{\tau}{F\cancel{\sin\theta}}$

$= \dfrac{24.0\ \text{N}\cdot\text{m}}{100\ \text{N}}$

$= 0.240$ m or 24.0 cm

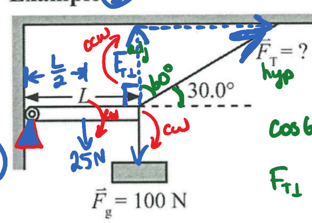


$\tau = F_\perp d$

$24 N\cdot m = 100N \cdot d$

$d = 0.24 m$

Example 3



$\cos 60° = \dfrac{F_{T\perp}}{F_T}$

$F_{T\perp} = F_T \cos 60°$

$$\Sigma \tau_{cw} = \Sigma \tau_{ccw}$$

$$(F_{cg}\cdot d_{cg}) + (F_g \cdot L) = (F_{T\perp}\cdot L)$$

$$25N\cdot\frac{\cancel{L}}{2} + 100N\cdot\cancel{L} = F_T\cos 60°\cdot\cancel{L}$$

$$\frac{112.5}{\cos 60°} = F_T\frac{\cos 60°}{\cos 60°}$$

$$F_T = 225\,N$$

so cg in middle

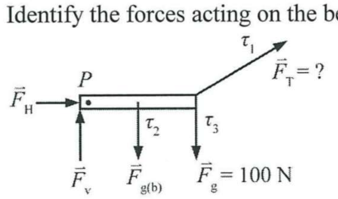
A 25.0 N uniform beam is attached to a wall by means of a hinge. Attached to the other end of this beam is a 100 N weight. A rope also helps to support the beam as shown in the diagram.

a) What is the magnitude of the tension in the rope?

Solution

This beam would rotate if the rope was not there. Therefore, use the second condition of equilibrium to solve the problem.

Identify the forces acting on the beam, and draw a free-body diagram.



The magnitudes of the parameters are as follows:

F_T = tension in supporting rope

F_g = weight of hanging mass

$F_{g(b)}$ = weight of beam

F_H = horizontal force exerted by wall

F_V = vertical force exerted by wall

NOTES

Identify the point of rotation—the hinge (point P) is the point of rotation.

Every force produces a torque except if the force acts on the point of rotation.

Identify these torques using the second condition of equilibrium.

$\Sigma\tau = 0$

$\tau_1 - \tau_2 - \tau_3 = 0$

$r_1F_1\sin\theta_1 - r_2F_2\sin\theta_2 - r_3F_3\sin\theta_3 = 0$

(Note: Since the length of the beam is unknown, let L be the length of the beam.)

$L\,F_T\ \sin 30° - \frac{1}{2}L(25.0\text{ N})\sin 90° - L(100\text{ N})\sin 90° = 0$

(Note: divide both sides of the equation by L and L will disappear.)

$F_T\ \sin 30° - \frac{1}{2}(25.0\text{ N})\sin 90° - (100\text{ N})\sin 90° = 0$

$0.50\ F_T - 12.5\text{ N} - 100\text{ N} = 0$

$F_T = \dfrac{12.5\text{ N} + 100\text{ N}}{0.50}$

$= 225\text{ N}$

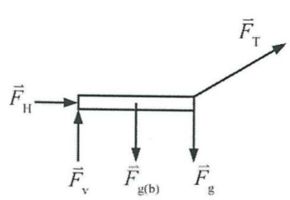
The tension in the rope is 225 N.

b) What are the vertical and horizontal forces that the wall exerts on the beam?

Solution

In order to solve for F_H and F_V, use the first condition of equilibrium. To do this, consider the beam to be a point.

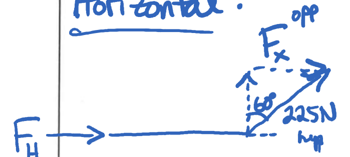
Draw a free-body diagram of the forces acting on the beam.



Note: the direction of $\vec{F}_H$ and $\vec{F}_V$ are not actually known

Break these forces into components if required:

Horizontal:



$\Sigma F_{rt} = \Sigma F_{left}$

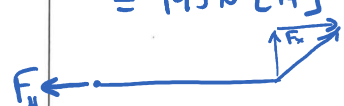
$F_H + F_x = 0$

$F_H = -F_x$

$= -225\text{N}(\sin 60°)$

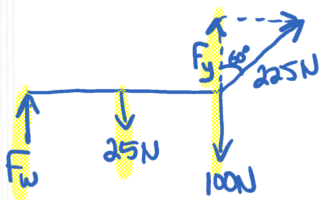
$= -195\text{N [rt]}$

$= 195\text{N [lf]}$



NOTES

Vertical:



$\cos 60° = \frac{F_y}{225}$

$F_y = 225 \cos 60°$

$\Sigma F_{up} = \Sigma F_{down}$

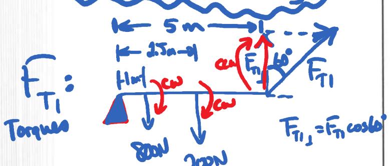
$F_w + F_y = 25 + 100$

$F_w + 225\cos 60 = 125$

$F_w = 12.5\text{ N [up]}$

positive so "up" was assumed correctly

F_{T1}: Torques



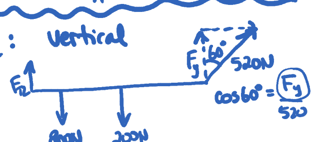
$\Sigma \tau_{cw} = \Sigma \tau_{ccw}$

$(800\text{N})(1\text{m}) + (200\text{N})(2.5\text{m}) = (F_{T1\perp})(5\text{m})$

$(800) + (500) = (F_{T1}\cos 60)(5)$

$F_{T1} = 520\text{ N}$

F_{T2}: vertical

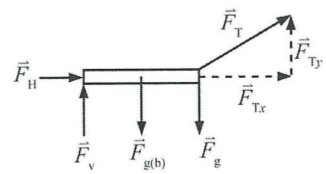


$\Sigma F_{up} = \Sigma F_{down}$

$F_{T2} + F_y = 800\text{N} + 200\text{N}$

$F_{T2} + 520\cos 60° = 1000\text{N}$

$F_{T2} = 740\text{N}$



$F_{Tx} = F_T \cos\theta$
$F_{Ty} = F_T \sin\theta$

Use the first co di i f equilibrium:

$$\sum \vec{F}_x = 0$$
$$F_{Tx} + F_H = 0$$
$$F_T \cos\theta = -F_H$$
$$(225\text{ N})\cos 30° = -F_H$$
$$(225\text{ N})(0.866) = -F_H$$
$$\therefore F_H = -195\text{ N}$$

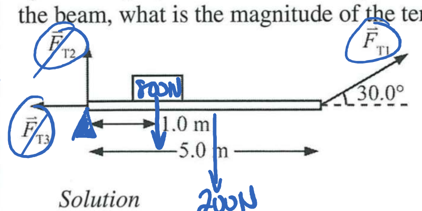
$\vec{F}_H$ is toward the left

$$\sum \vec{F}_y = 0$$
$$F_{Ty} + F_V - F_{g(b)} - F_g = 0$$
$$F_T \sin\theta - F_{g(b)} - F_g + F_V = 0$$
$$(225\text{ N})\sin 30° - 25\text{ N} - 100\text{ N} + F_V = 0$$
$$112.5\text{ N} - 25\text{ N} - 100\text{ N} + F_V = 0$$
$$F_V = 12.5\text{ N}$$

$\vec{F}_V$ is directed upward.

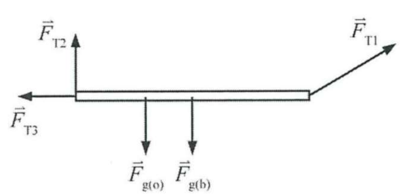
Example 4

A uniform beam 5.0 m long has a weight of 200 N on it and is suspended by three ropes, as shown in the diagram. If an 800 N object is placed on the beam, what is the magnitude of the tension in each of the ropes?



Solution

Draw a free-body diagram.



F_{T1}, F_{T2}, F_{T3} are the magnitudes of the tension in the ropes.

$F_{g(o)}$ = magnitude of weight of the object

$F_{g(b)}$ = magnitude of weight of the beam

F_{T3}: horizontal

$\Sigma F_{lt} = \Sigma F_{rt}$

$F_{T3} = F_x$

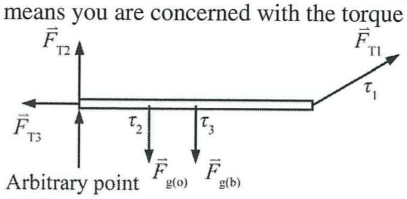
$= 520\text{N}(\sin 60°)$

$= 450\text{N}$

NOTES

Begin solving this problem by using the second condition of equilibrium to identify the torques involved. But in order to identify the torques, there needs to be a point of rotation. In this case, there is no obvious point of rotation. The point of rotation in this case is arbitrary—in other words, choose where place to place the point of rotation. Because there are two unknown forces at the left end of the beam, treat this point as the point of rotation.

Remember: There is no torque associated with forces acting on the point of rotation. Therefore, treating this point as the point of reference means you are concerned with the torque due to F_{T1}.

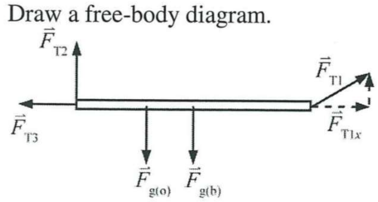


Use the second condition of equilibrium to find the magnitude of F_{T1}.

$$\sum \tau = 0$$
$$\tau_1 - \tau_2 - \tau_3 = 0$$
$$r_1 F_1 \sin\theta_1 - r_2 F_2 \sin\theta_2 - r_3 F_3 \sin\theta_3 = 0$$
$$(5.0\ \text{m})F_{T1}(\sin\ 30°) - (1.0\ \text{m})(800\ \text{N})(\sin 90°) - (2.5\ \text{m})(200\ \text{N})(\sin 90°) = 0$$
$$(2.5\ \text{m})F_{T1} - 800\ \text{N}–500\ \text{N}\cdot\text{m} = 0$$
$$F_{T1} = \frac{800\ \text{N}\cdot\text{m} + 500\ \text{N}\cdot\text{m}}{2.5\ \text{m}} = 520\ \text{N}$$

Now use the first condition of equilibrium to find the magnitudes of F_{T2} and F_{T3}.

Draw a free-body diagram.



$F_{T1x} = F_{T1}\cos\theta$ $\qquad$ $F_{T1y} = F_{T1}\sin\theta$

Use the first condition of equilibrium.

$$\Sigma \vec{F}_x = 0$$
$$F_{T1x} - F_{T3} = 0$$
$$F_{T1}\cos\theta = F_{T3}$$
$$(520\ \text{N})\cos 30° = F_{T3}$$
$$\therefore \quad F_{T3} = 450\ \text{N}$$

$$\Sigma \vec{F}_y = 0$$
$$F_{T1y} + F_{T2} - F_{g(o)} - F_{g(b)} = 0$$
$$F_{T1}\sin\theta - F_{g(o)} + F_{g(b)} + F_{T2} = 0$$
$$(520\ \text{N})\sin 30° - 800\ \text{N} - 200\ \text{N} + F_{T2} = 0$$
$$260\ \text{N} - 800\ \text{N} - 200\ \text{N} + F_{T2} = 0$$
$$F_{T2} = 740\ \text{N}$$

The tension in the three ropes have magnitudes of 520 N, 740 N, and 450 N.

$\tau = F_{\perp} d$

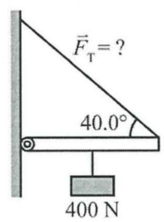
PRACTICE EXERCISES

Formulas: $\tau = rF$ $\cancel{\Sigma\tau = 0}$ $\Sigma\tau_{cw} = \Sigma\tau_{ccw}$

not equilibrium

1. If the torque needed to loosen a lug nut holding the wheel of a car is 45 N·m and you are using a wheel wrench that is 35 cm long, what is the magnitude of the force you exert perpendicular to the end of the wrench?

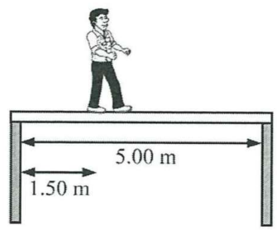
2. A beam of negligible mass is attached to a wall by means of a hinge. Attached to the centre of the beam is a 400 N weight. A rope also helps to support this beam as shown in the illustration.



a) What is the magnitude of the tension in the rope?

b) What are the magnitudes of the vertical and horizontal forces that the wall exerts on the beam?

3. A 650 N student stands on a 250 N uniform beam that is supported by two supports as shown in the diagram.

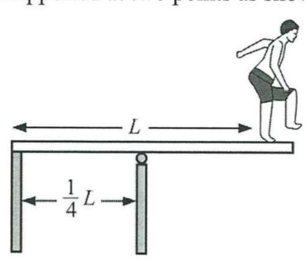


If the supports are 5.00 m apart and the student stands 1.50 m from the left support,

a) what is the magnitude of the force that the right support exerts on the beam?

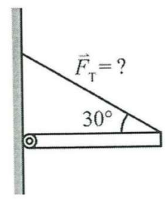
b) what is the magnitude of the force that the left support exerts on the beam?

4. A uniform 400 N diving board is supported at two points as shown in the illustration.



If a 75.0 kg diver stands at the end of the board, what are the forces acting on each support?

5. Find the tension in the rope supporting the 200 N hinged uniform beam shown in the illustration.



6. Find the tension in the rope supporting the 200 N hinged uniform beam shown in the illustration.