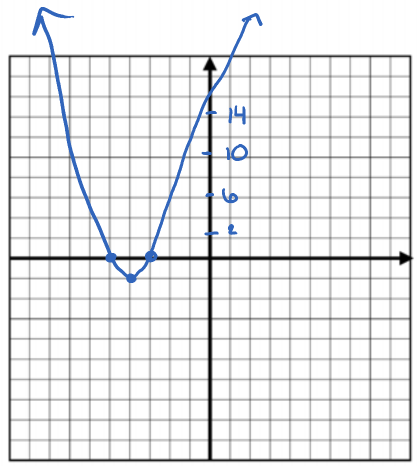
7.5 Solving Quadratics by Factoring

Solving Quadratic Equations by Factoring [7.5]

Let's start slow. What do you need to know to graph: $y = x^2 + 8x + 15$

- x-int
- y-int
- vertex
- direction

Can you do it?



U ← $x^2 + 8x + 15$ → y-int

$(x+3)(x+5)$

$x = -3, -5$

Vertex: ① $\frac{-3 + (-5)}{2}$, $\frac{-8}{2} = -4$

② $y = x^2 + 8x + 15$

$y = (-4)^2 + 8(-4) + 15$

$y = -1$

③ V $(-4, -1)$

What patterns do you notice with the following numbers? Perfect squares

4	9	16	25	36	49	64	81	100	x^2	y^2	♪^{2}
$2 \cdot 2$	$3 \cdot 3$	$4 \cdot 4$	$5 \cdot 5$	$6 \cdot 6$	$7 \cdot 7$	$8 \cdot 8$	$8 \cdot 8$	$10 \cdot 10$	$x \cdot x$	$y \cdot y$	♪ · ♪

What do you notice about the following quadratic? Perfect square trinomial

$x \cdot x$ $\quad$ $6 \cdot 6$

$$x^2 + 12x + 36 = 0$$

$(x+6)(x+6)$

Can you show me another way to represent it?

$(x+6)^2$

$\rightarrow x = ?$

Using all the tricks you've re-visited so far. Solve the following equation.

* I know it's tricky… do it anyways!

so it's equal to zero, then factor

$$9x^2 + 42x = -49$$

$$9x^2 + 42x + 49 = 0$$

$$(3x+7)(3x+7)$$

$$3x+7=0$$

$$3x=-7$$

$$x = \frac{-7}{3}, \frac{-7}{3}$$

How do you know your answer is right?

- Replace x with $-\frac{7}{3}$. If right side = left side > you're good
- Factor brackets using FOIL to check.

Making it work in reverse. What quadratic equation could have the roots – 6 and 8?

$ax^2+bx+c=0$

x-int, zeros,

$x = -6$ (+6, +6) $x = 8$ (−8, −8)

$x+6=0$ $x-8=0$

$(x+6)=0$ $(x-8)=0$

$(x+6)(x-8)=0$ → Use Foil to expand

$$(x+6)(x-8)=0$$

$$x^2 - 8x + 6x - 48 = 0$$

$$x^2 - 2x - 48 = 0$$

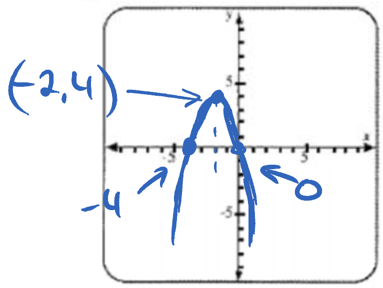
$$\boxed{x^2-2x-48=0 \quad \text{OR} \quad y = x^2-2x-48}$$

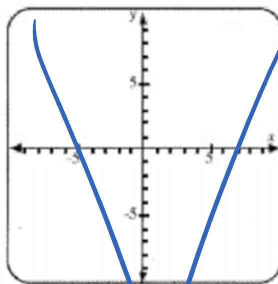
Time to work on the assignment – then a quick write to show what you know

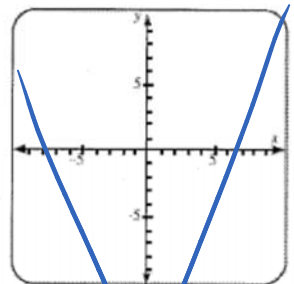
Check In

Pg 14

1 > For each of the graphs below please state:







Vertex: $(-2,4)$

Max/Min:

Axis of Sym: $x = -2$

Y-int: 0

Domain: $x \in \mathbb{R}$

Range: $y \leq 4$

2 > Solve for x: $-2x^2 + 2x + 40 = 0$ ← yint

$$-2(x^2 - x - 20) = 0$$

$$-2(x-5)(x+4) = 0$$

$$\left.\begin{array}{l} x-5=0 \rightarrow x=5 \\ \text{or} \\ x+4=0 \rightarrow x=-4 \end{array}\right\} \text{x-intercepts}$$

axis of sym

$$\frac{5+-4}{2} = \frac{1}{2} = 0.5 \qquad x=0.5 \leftarrow \text{plug into eq}^n\text{, solve for y}$$

vertex $(0.5, y)$

pg 15

3 > Solve this quadratic equation by factoring. (Hint: can multiply by something first)

$$(0.125x^2 - 0.875x = -1.5)\times 8$$

$$x^2 - 7x = -12$$

$$x^2 - 7x + 12 = 0$$

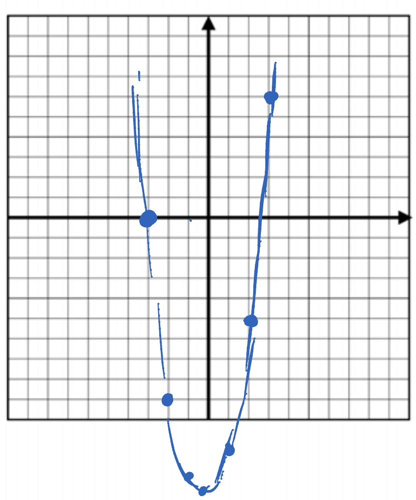
$$(x-4)(x-3) = 0$$

x-int = 4 and 3

(4,0) (3,0)

→ x=? → x-int.

4 > Solve by graphing: $2y^2 + y - 15 = 0$



x	y
-3	0
-2	-9
-1	-14
0	-15
1	-12
2	-5
3	6

crosses x-axis at
-3 and 2.5

7.6 (1) Vertex Form

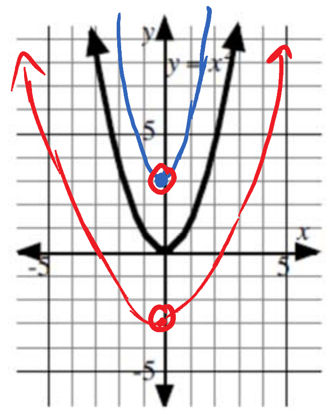
Vertex Form of a Quadratic Function [7.6]

Quadratic functions can be written in **general form** or **vertex form**.

General Form: $y = ax^2 + bx + c$ Vertex Form: $\boxed{y = a(x - p)^2 + q}$

Factored Form : $y = a(x-r)(x-s)$

Investigation #1 **Analyzing the Graph of $y = x^2 + q$**



The graph of $y = \boxed{f(x) = x^2}$ is shown.

a) Write an equation which represents each of the following:

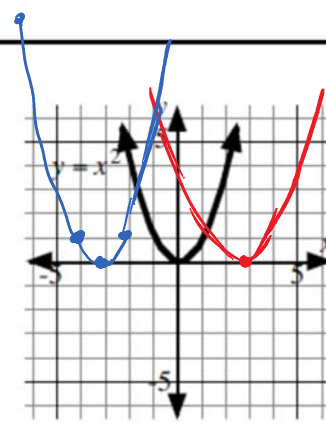
put it into vertex form →

- $y = x^2+3$
  - $y = f(x) + 3$
  - $y = 1(x-0)^2 + 3$
  - vertex : (0, 3)
- $y = x^2-3$
  - $y = f(x) - 3$
  - $y = 1(x-0)^2 - 3$
  - (0, −3)

b) ~~Use a graphing calculator to sketch~~ $y = f(x) + 3$ and $y = f(x) - 3$ on the grid.

What do we know? q represents the "y" coordinate of the vertex. It moves the vertex up or down

Investigation #2 **Analyzing the Graph of $y = (x - p)^2$**



The graph of $y = f(x) = x^2$ is shown.

a) Write an equation which represents each of the following:

- $y = f(x + 3)$
  - vertex form → $y = (x+3)^2 + 0$
  - vertex (−3, 0)

x	y				
0	9	-2	1	-6	9
2	25	-4	1	-3	0

- $y = f(x - 3)$
  - $y = (x-3)^2 + 0$
  - vertex (3, 0)

b) Use a graphing calculator to sketch $y = f(x + 3)$ and $y = f(x - 3)$ on the grid.

What did we learn? p represents the "x" of the vertex except the <u>sign changes</u>. It moves the graph right or left.

Putting it all together. How would you explain Vertex Form to someone else?

+a

-a

$$y = a(x-p)^2 + q$$

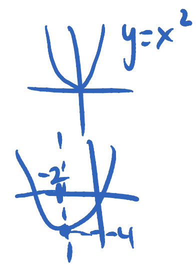
shift up or down

shift graph left or right

$(p,q) =$ vertex

Show me you understand:

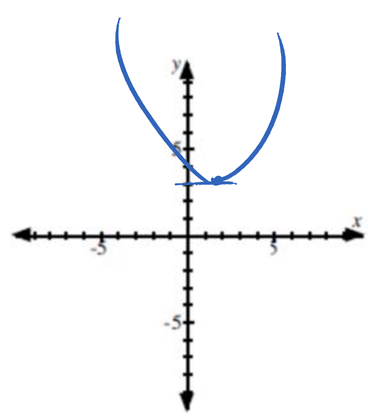
Function	*Equation Representing Function*	*Vertex*	*Max/Min Value*	*Equation of Axis of Symmetry*
$y = f(x)$	$y = x^2$	$(0, 0)$	min, 0	$x = 0$
$y = f(x+2) - 4$	$y = (x+2)^2 - 4$	$(-2, -4)$	min, -4	$x = -2$
$y = f(x-p) + q$	$y = (x-p)^2 + q$	(p, q)	min, q	$x = p$



Time to show off:

Consider the graph of the function $f(x) = (x-2)^2 + 3$.

a) Without using a graphing calculator, sketch the graph on the grid.



b) State the coordinate of the vertex. $(2, 3)$

c) State the maximum or minimum value of the function. 3

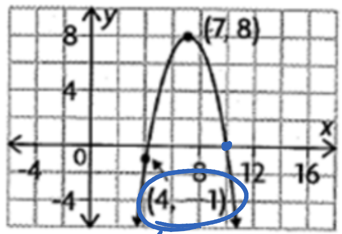
d) State the domain and range of the function.

D: $x \in R$

R: $y \geq 3$

$y = a(x-p)^2 + q$

Example: Determine the quadratic function corresponding to this parabola.



vertex: (7, 8) ← plug into formula

$y = a(x-7)^2 + 8$

need to find "a" .... how?

point on line
$(4, -1)$
x   y

plug in point for x and y

$-1 = a(4-7)^2 + 8$

$-1 = 9a + 8$

$-9 = 9a$

$-1 = a$

Final formula: $y = -1(x-7)^2 + 8$

Pg417 # 1ac, 2ac, 4

Project due

Jan 6 ☺

7.6 (2) Vertex Form   Happy New Year!

Vertex Form of a Quadratic Function – Day 2 [7.6]

Vertex Form. What does that look like again?

$$y = a(x-p)^2 + q$$

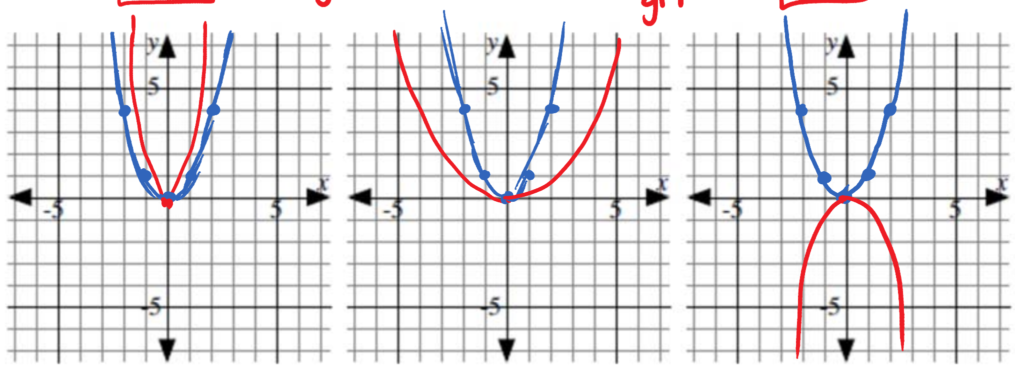
q: shift up or down, max or min of y.

p: shift right or left, axis of symmetry

So what about "a"?

normal $y = x^2$, $a = 1$

$a > 1$ → taller narrower graph $0 < a < 1$ → short wide graph $a < 1$ - inverted



vertex form $y = (x - p)^2 \pm q$

Graphing with the Method of Differences

M.O.D. : 1, 3, 5

Graph: $y = (x+3)^2 - 2$

$y = (x - {-3})^2 + {-2}$

→ vertex $(-3, -2)$

① plot vertex

② $a = 1$ ← "+" so opens up

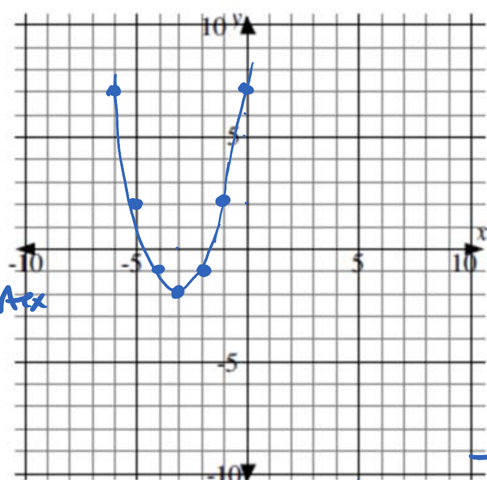
a

$1 \cdot 1$ → over 1, up 1 from vertex

$1 \cdot 3$ → over 1, up 3

$1 \cdot 5$ → over 1, up 5

always measure from the last point



D: $x \in R$

R: $y \geq -2$, $y \in R$

axis of symm.

$x = -3$

min value

$y = -2$

$y=m(x-p)^2+q$

Sketch the graph of the quadratic function $y = -2(x + 4)^2 + 10$.

$p=-4 \quad q=10$

State the domain & range of the function.

Vertex $(-4,10)$

$a=-2 \rightarrow$ opens down

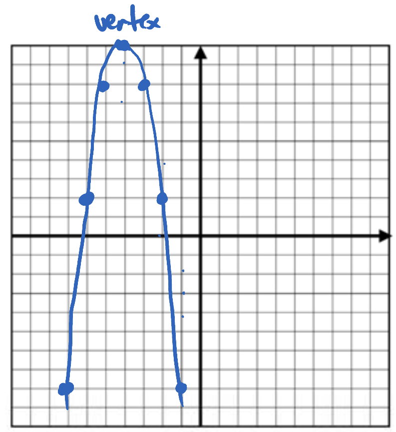
MOD

a

$-2 \cdot 1 = -2$ over 1, down 2

$-2 \cdot 3 = -6$ over 1, down 6

$-2 \cdot 5 = -10$ over 1, down 10



D: $x \in R$

R: $y \leq 10$, $y \in R$

Applying it all to Real Life...

x t, y h

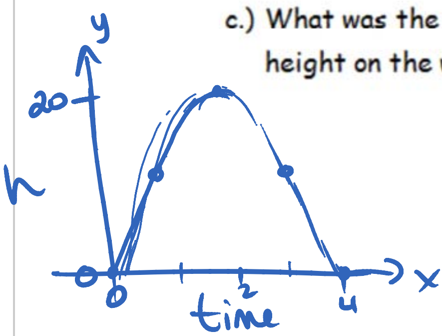
$V = (2,20)$

A soccer ball is kicked from the ground ($h=0$). After 2s the ball reaches its maximum height of 20m. It lands on the ground at 4s. $\rightarrow$ gives another pt $(4,0)$

a.) Determine the quadratic function that models the height of the kick.

b.) Determine any restrictions that must be placed on the domain and range of the function.

c.) What was the height of the ball at 1s? When was the ball at the same height on the way down?



a) $(p,q) = (2,20)$ vertex

$y = a(x-p)^2 + q$

$y = a(x-2)^2 + 20$

use the pt $(4,0)$ to find a value for "a"

$0 = a(4-2)^2 + 20$

$\frac{-20}{4} = \frac{a(2)^2}{4}$

$-5 = a \rightarrow \boxed{y = -5(x-2)^2 + 20}$

b) D: $0 \leq x \leq 4$, $x \in R$

R: $0 \leq y \leq 20$, $y \in R$

c) $y = -5(x-2)^2 + 20$

$h = -5(1-2)^2 + 20$

math way

$15 = -5(x-2)^2 + 20 \rightarrow$ solve for x

c) $y = -5(x-2)^2 + 20$

$h = -5(1-2)^2 + 20$

$= -5 + 20$

$h = 15m$

math way

$15 = -5(x-2)^2 + 20 \rightarrow$ solve for x

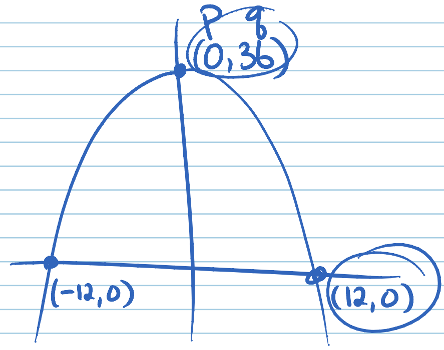
or

logic way $\rightarrow$ 15m on way down is symmetrical on parabola

so $t = 3$

7.6 Homework

(11)



$y = a(x-p)^2 + q$

$y = a(x-0)^2 + 36$

$y = ax^2 + 36$

$0 = a(12)^2 + 36$

$-36 = 144a$

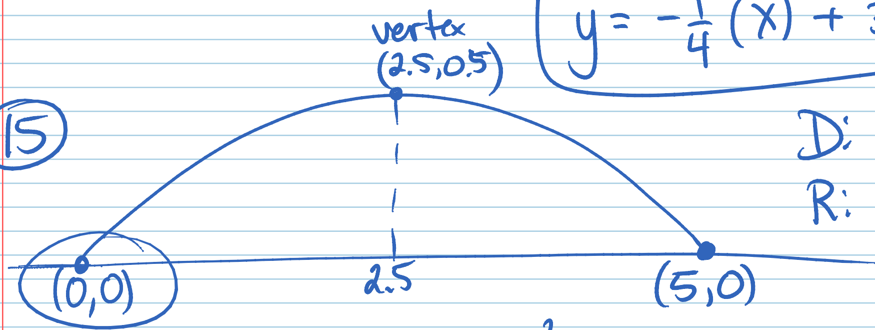
$\frac{-36}{144} = a$

$-\frac{3}{12} = a$

$-\frac{1}{4} = a$

$y = -\frac{1}{4}(x)^2 + 36$

(15)



D: $0 \le x \le 5$

R: $1 \le y \le 1.5$

or

$0 \le y \le 0.5$

$y = a(x-2.5)^2 + 0.5$

use any point on graph (0,0)

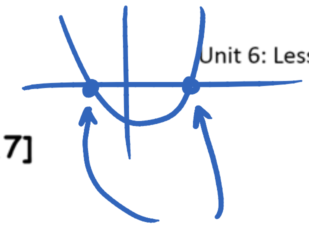
$0 = a(0-2.5)^2 + 0.5$

$-0.5 = (-2.5)^2(a)$

$\frac{-0.5}{6.25} = a$

$-0.08 = a$

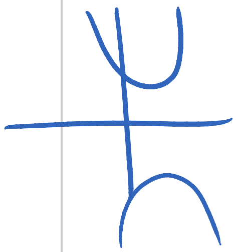
$y = -0.08(x-2.5)^2 + 0.5$



Using the Quadratic Formula [7.7]

The Quadratic Formula

solutions, zeros, where crosses x-axis



The quadratic equation $ax^2 + bx + c = 0, a \neq 0$ has the roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Before we get started, a few helpful hints:
- use brackets when replacing a variable
- if the radical is negative then there is no solution ∄
- show your steps

Example 1: Solve the quadratic equation $\boxed{4x^2 - 3 = 7x}$. Give an exact answer and an approximate answer to 3 decimal places.

① put eqⁿ in general form: $4x^2 - 7x - 3 = 0$

$ax^2 + bx + c = 0$

$\therefore a = 4$
$b = -7$
$c = -3$

② plug a, b, c into quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

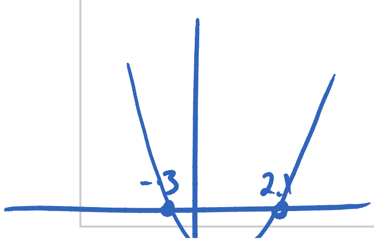
$$= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(4)(-3)}}{2(4)}$$

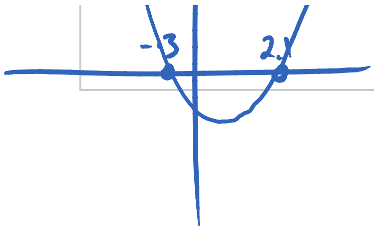
$$= \frac{7 \pm \sqrt{49 + 48}}{8}$$

$$= \frac{7 \pm \sqrt{97}}{8} \implies$$

exact answers

$$\boxed{x = \frac{7 + \sqrt{97}}{8} \text{ or } \frac{7 - \sqrt{97}}{8}}$$





$$= \frac{7 \pm \sqrt{97}}{8} \Rightarrow \boxed{x = \frac{7+\sqrt{97}}{8} \text{ or } \frac{7-\sqrt{97}}{8}}$$

rounded answers $\rightarrow x = 2.106$ or -0.356

Try this one: Solve the quadratic equation $2x^2 + 8x - 5 = 0$

a b c

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(8) \pm \sqrt{(8)^2 - 4(2)(-5)}}{2(2)}$$

$$= \frac{-8 \pm \sqrt{64+40}}{4}$$

$$= \frac{-8 \pm \sqrt{104}}{4}$$

$$= \frac{-8 \pm 2\sqrt{26}}{4}$$

$$= \frac{-8}{4} \pm \frac{2\sqrt{26}}{4}$$

$$= \boxed{-2 \pm \frac{\sqrt{26}}{2}}$$

$$= \boxed{\frac{-4 \pm \sqrt{26}}{2}}$$

104 → 2, 52; 52 → 2, 26; 26 → 2, 13

$$\sqrt{104} = \sqrt{2^2 \cdot 2 \cdot 13} = 2\sqrt{26}$$

$$\frac{4+6}{2} = \frac{10}{2} = 5$$

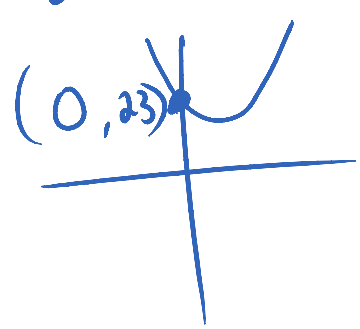
$$= \frac{4}{2} + \frac{6}{2} = 2 + 3 = 5$$

Solve the quadratic equation $x^2 + 9x + 23 = 0$. Draw a really quick sketch to demonstrate what you found out.

$a = 1$
$b = 9$
$c = 23$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(9) \pm \sqrt{(9)^2 - 4(1)(23)}}{2(1)}$$

$$= \frac{-9 \pm \sqrt{-11}}{2} \leftarrow \text{no solution}$$



Working with a partner, write the steps to solve this problem.

A store rents an average of 750 video games each month at a current rate of \$4.50. The owners of the store want to raise the rental rate to increase the revenue to \$7000 per month. However, for every \$1 increase, they know that they will rent 30 fewer games each month. The following function relates the price increase, p, to the revenue, r.

$$(4.5+p)(750-30p)=r$$ ← want 7000

Can the owners increase the rental rate enough to generate revenue of \$7000 per month?

① replace r with \$7000 $(4.5+p)(750-30p)=7000$

② FOIL $3375-135p+750p-30p^2=7000$

③ Write it in general form → $\underbrace{-30}_{a}p^2+\underbrace{615}_{b}p\underbrace{-3625}_{c}=0$

④ Use the quadratic formula to find p

$$p=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(615)\pm\sqrt{(615)^2-4(-30)(-3625)}}{2(-30)}$$

$$=\frac{-615\pm\sqrt{-56775}}{-60}$$

∴ no solution for p that will allow a revenue of \$7000

Practice on next 2 pages of package.

7.7 Worksheet

7.7 Solving Quadratic Equations Using the Quadratic Formula

Keep in Mind

- The roots of a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$, can be determined using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- You can use this formula to determine the roots, if they exist, of any quadratic equation, even if it is not factorable.
- The quadratic formula will give you an exact value for the solution.
- If the radicand, $b^2 - 4ac$, simplifies to a perfect square, then the equation can be solved by factoring.
- If the value of the radicand is negative, then the equation has no real solution.

2. Suppose you were to solve these equations using the quadratic formula. What values of a, b, and c would you use in each case?

a) $3x^2 - 2x + 1 = 0$

b) $-2(x - 1)^2 - 1 = 0$

4. Solve each quadratic equation. Identify any equations that do not have real roots. Otherwise, give an exact answer.

a) $2x^2 - x - 3 = 0$

c) $16x^2 + 8x + 3 = 0$

b) $5x^2 = 6x + 2$

$5x^2 - 6x - 2 = 0$

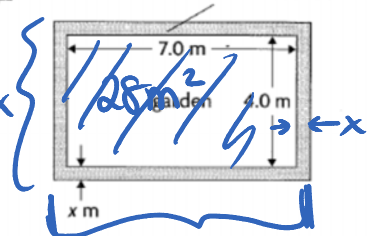
a b c

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(5)(-2)}}{2(5)}$$

d) $-2x^2 + 8x = 3$

$$x = \frac{6 \pm \sqrt{76}}{2(5)} = \frac{6 \pm \sqrt{4 \cdot 19}}{10} = \frac{6 \pm 2\sqrt{19}}{10} = \frac{3 \pm \sqrt{19}}{5}$$

5. A landscaper is designing a rectangular garden, as shown. She has enough crushed rock to cover an area of 10.0 m² and wants to make a uniform border around the garden. How wide should the border be, if she wants to use all the crushed rock?



$$(7+2x)(4+2x) - 28 = 10m^2$$

$$x = \frac{-22 \pm \sqrt{644}}{8} = 0.4m \text{ or } \cancel{-5.9m}$$

a b c

$-1.89t^2+5t+7.5=0$

6. On Mars, a ball thrown from the top of a spacecraft 6.5 m high could be modelled by $h(t) = -1.89t^2 + 5t + 7.5$. On Neptune, a ball thrown from the top of the same spacecraft could be modelled by $h(t) = -7.0t^2 + 5t + 7.5$. In these equations, h is the height in metres and t is the time in seconds. How much earlier would a ball fall to the base of the spacecraft on Neptune than on Mars? Give your answer to the nearest hundredth of a second.

$t_m = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = 3.71397$

$t_N = 1.45212$

on Mars: $t =$ 3.71 (Choose the positive root.)

on Neptune: $t =$ 1.45

The ball would fall to the base of the spacecraft 2.26s earlier on Neptune.

7. Suppose a pebble were to fall from a 200 m cliff to the water below. The height of the stone, $h(t)$, in metres, after t seconds can be represented by the function $h(t) = -4.9t^2 + 3t + 200$. How long would the stone take to reach the water, to the nearest tenth of a second? Show your work.

$$t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

2. a) $a = 3, b = -2, c = 1$ b) $a = -2, b = 4, c = -3$

4. a) $x = 1.5$ or $x = -1$

b) $x = \frac{3 - \sqrt{19}}{5}$ or $x = \frac{3 + \sqrt{19}}{5}$

c) no real roots

d) $x = \frac{4 - \sqrt{10}}{2}$ or $x = \frac{4 + \sqrt{10}}{2}$

5. $\doteq 0.4$ m

6. on Mars: $t =$ 3.713...

on Neptune: $t =$ 1.452...

The ball would fall to the base of the spacecraft 2.26 s earlier on Neptune.

7. $\doteq 6.7$ s

Ch 7 Review Notes

Thursday, January 09, 2014
1:52 PM

Big Idea

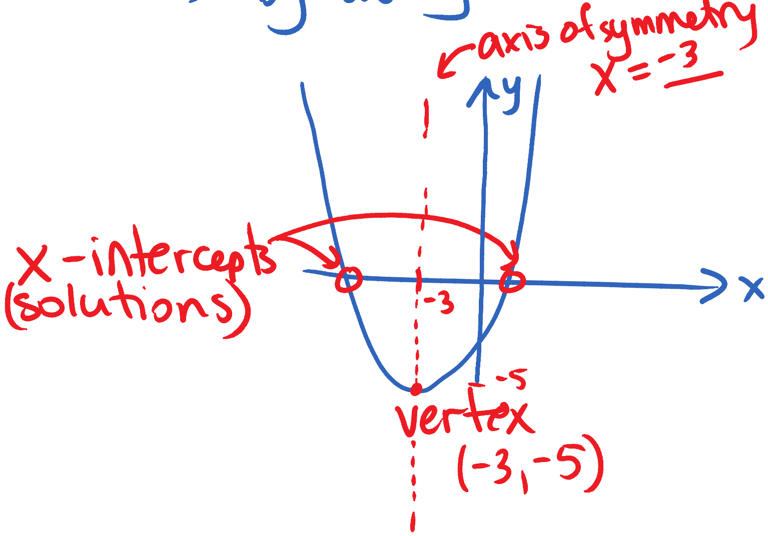
Quadratic Equation $ax^2+bx+c=0$ ← y-intercept

$x =$ solution, zero, root, where crosses the x-axis

Ways to find the solutions:

Graphing

→ by using table of values



x	y
-3	
-2	
-1	
0	
1	
2	
3	

Domain: $x \in R$

Range: $y \geq -5$ (min)
or
$y \leq$ max

y-intercept → find by setting $x = 0$
x-intercept → find by setting $y = 0$

→ by factoring

ex $x^2 + x - 12 = 0$ (+ above $+x$, × above -12)

$(x+4)(x-3) = 0$

$\underbrace{x = -4, \ x = 3}_{\text{solutions / x-intercepts}}$

Factored form:

$$y = a(x-r)(x-s)$$

solutions/x-intercepts

- $a > 0$
- $a < 0$
- $a > 1$ tall/narrow
- $0 < a < 1$ short/wide

→ by using vertex form

shifts up/down

$$y = a(x-p)^2 + q$$

same as before

vertex (p, q)

shifts right/left

Quadratic Formula

solution, x-intercept, root → $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

if "−" under radical then no solution, never crosses x-axis