7.1 Exploring Quadratic Relations - Key

Exploring Quadratic Relations [7.1]

Quadratic Equation

A **quadratic equation** is an equation which can be written in the form

degree = 2 always!

$ax^2 + bx + c = 0$, where $a, b, c \in R$, and $a \neq 0$.

that's why $a \neq 0$

y-int ($x=0$)

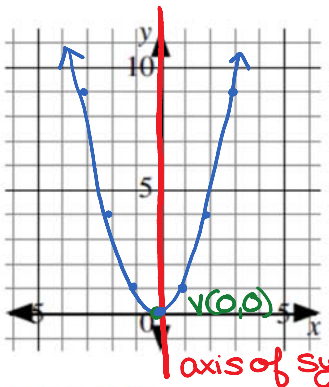
Analyzing the Graph of the Function with Equation $y = x^2$

- Graph the function with equation $y = x^2$ by completing the table of values. Join the points with a smooth curve. The graph of this function is called a parabola.

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

$(-3)^2$ $(-2)^2$ $(-1)^2$ $(0)^2$ $(1)^2$ $(2)^2$ (3)

(−3,9) (−2,4) (−1,1) (0,0) (1,1) (2,4) (3,9)



- The **axis of symmetry** is the "mirror" line which splits the parabola in half. State the equation of the axis of symmetry for this parabola.

$x = 0$

- The **vertex** of a parabola is where the axis of symmetry intersects the parabola. The vertex can represent a minimum point or maximum point depending on whether the parabola opens up or down.

Label the vertex (V) on the graph and state its coordinates.

V (0,0) → "Turning point"

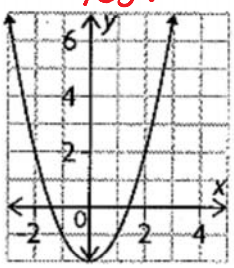
Some Key Ideas

- See x^2 → think ∪ or ∩ "Parabola"
- If "a" is positive → graph is happy and opens up ∪
  If "a" is negative → graph is sad and opens down ∩

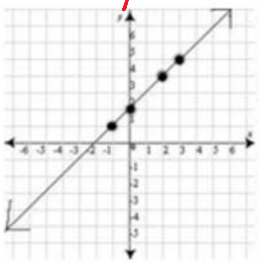
↑↓ or ↓↑ $y=x^2$

Example 1: Which of the following are **Quadratic relations**?

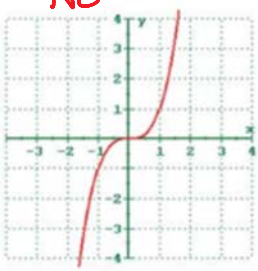
Yes!



No - $y=mx+b$

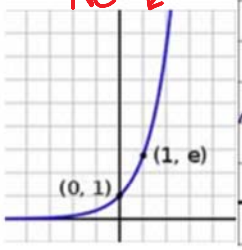


No

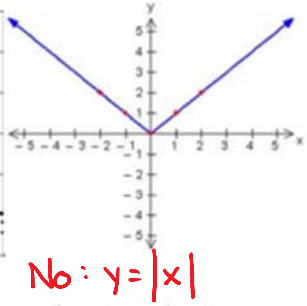


Yes! Not happy ∴ $-x^2$

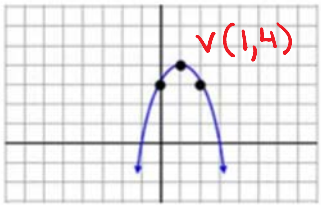
No ↓



Nope!



No: $y=|x|$



$(-x^2)$

Yes!

- x^2 is highest degree
- no $\sqrt{x}$
- no $\frac{1}{x}$

Example 2: Which of the following are quadratic relations?

Y a.) $y = 3x^2 + 7x - 2$ y-int: (0,-2) (x=0)

N b.) $y = x^2 + \sqrt{x}$

Y c.) $y = 25 - 9x^2$ y-int: (0,25)

Y d.) $y = 7 - 5x^2$ y-int: (0,7)

Y e.) $y = 2x^2 + 11 - 4x$ y-int: (0,11)

$y = 2x^2 - 4x + 11$

N f.) $y = \frac{1}{4x^2-9x+12}$

N g.) $y = 2x^3 + 6x - 1$

Y h.) $y = (x+2)^2 - 7$

$y=(x+2)^2-7$

$y=(0+2)^2-7$

$y = 4-7$

$y=-3$

N i.) $y = 2x - 8$

$y=mx+b$ (line)

Part 2: Determine the y-intercept for each quadratic relation above. ☐

x=0

Example 3: Does the parabola open up [+a] or down [-a]?

a.) $y = -(2x+5)^2 - 8$ ↓↑

b.) $y = 7 + 2x^2$ ↑↓

c.) $y = -7 + 12x + 3x^2$ ↓↑

d.) $y = -\frac{1}{2}x^2 - 2x + 7$ ↓↑

7.2 Properties of Graphs of Quadratic Functions - Key

Properties of Graphs of Quadratic Functions [7.2]

Warm Up:

This table of values lists points in a quadratic relation.

a) What is the y-intercept of the parabola? x=0 (0, -2)

Is this the highest or lowest point on the parabola? (-1, -3) vertex

b) Without graphing, predict the direction in which the parabola opens. Explain how you know.

opens up → from vertex, y values are increasing

x	y
−3	1
−2	−2
−1	−3
0	−2
1	1
2	6

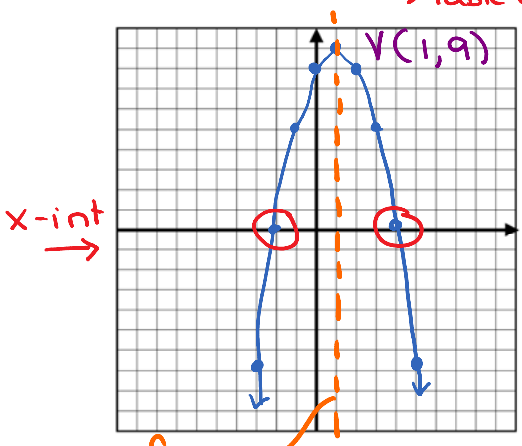
x=0 → Vertex

Example 1:

Determine the y-intercept (x=0), any x-intercepts (y=0), the equation of the axis of symmetry, the coordinates of the vertex, and the domain and range of the function

$f(x) = -x^2 + 2x + 8$. Sketch the graph. ↳ Table of Values

Domain: D_x Range: R_y



x	y
-3	-7
-2	0
-1	5
0	8
1	9
2	8
3	5
4	0

always start with these #s.

$y = -x^2 + 2x + 8$
$y = -(-3)^2 + 2(-3) + 8$
$y = -(9) - 6 + 8$
$y = -9 - 6 + 8$
$y = -7$

→ once you understand your calculator will do this.

→ y int > x=0
$y = -(0)^2 + 2(0) + 8$
$y = 8$

x-int > y=0
(-2,0) ; (4,0)

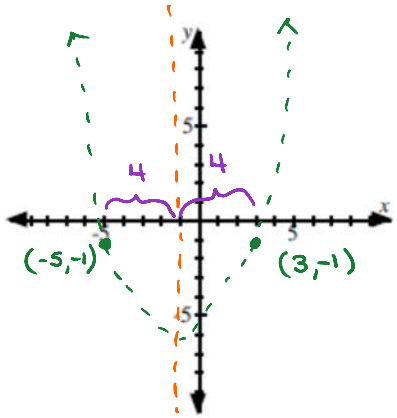
Domain: $x \in \mathbb{R}$
Range: $y \leq 9$

Example 2:

$x = ?$ (x, y) (x, y)

Determine the equation of the axis of symmetry if $(-5, -1)$ *and* $(3, -1)$ are located on the parabola.

Notice they have the same y value → use midpoint formula



$$\frac{x_1 + x_2}{2}$$

$$\frac{-5+3}{2}$$

$$\frac{-2}{2}$$

$$= -1$$

$\therefore x = -1$

Example 3:

blah, blah, blah...

Some children are playing at the local splash pad. The water jets spray water from ground level. The path of water from one of those jets forms an arch that can be defined by the function

$h = 0$

open down, Quadratic!, y-int at (0,0)

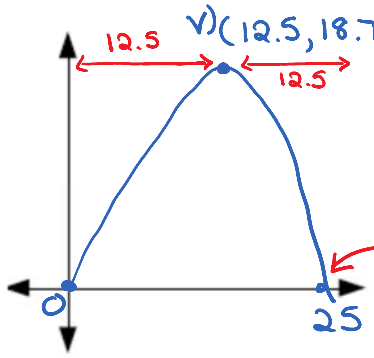
$$f(x) = -0.12x^2 + 3x + 0$$

where x represents the horizontal distance from the opening in the ground in feet and $f(x)$ is the height of the sprayed water, also measured in feet. What is the maximum height of the arch of water, and how far from the opening in the ground can the water reach? $y = 0$

No negatives
Real Place, Whole #s

x	y
0	0
2	5.5
4	10.08
6	13.62
8	16.32
10	18
12	18.72
13	18.72
14	18.48

Use calculator!
$y = -0.12(2)^2 + 3(2)$



$12.5 + 12.5 = 25$

max height = 18.75 ft.
Water reaches 25 ft.

$V(12.5, 18.75)$ → same! Vertex must be in the middle

* starts going back down