

## 6.1 Representing Patterns

Prescribed Learning Outcomes (PLO'S):

- Write a linear expression representing a given pictorial, oral, or written pattern
- Write a linear equation to represent a given context
- Describe a context for a given linear equation
- Solve using a linear equation, a given problem that involves pictorial, oral, and written linear patterns
- Write a linear equation representing the pattern in a given table of values and verify the equation by substituting values from the table
- Determine, by substitution, whether a given rational number is a solution to a given linear equation

Terminology

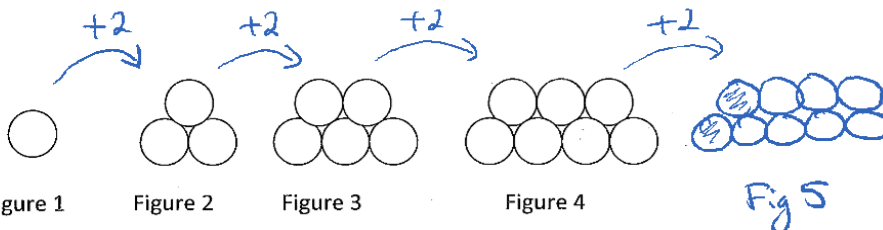
$$\begin{array}{c} +5=d \\ \curvearrowright \\ 0, 5, 10, 15, \dots \end{array}$$

**Linear patterns:** a sequence of numbers in which the pattern only involves addition or subtraction.**Common Difference  $d$ :** is the difference between any two consecutive numbers in a linear pattern.

$$d=5$$

## Observing a Pattern

Draw the Figure 5 by continuing the pattern shown below. How is the number of circles related to the figure number? Show the relationship in a table.



input Figure #	output # of circles
1	1
2	3
3	5
4	7
5	9

What do you notice as we move from figure to figure? What is changing? How is it changing? Look at the table. Do you see any patterns?

$$d=2$$

(Fig #)(d)	# of circles
1 · 2 = 2	1
2 · 2 = 4	3
3 · 2 = 6	5
4 · 2 = 8	7
5 · 2 = 10	9

Make a new table of values. This time multiply the figure # by the change in the number of circles and put this on the left-hand side of your table. The right-hand side should stay the same. What can you see now?

$$d \cdot \text{input} \dots = \text{output}$$

Write an equation relating:

the figure # (call it "N") and

the # of circles (call it "C")

$$\begin{array}{l} 2 \cdot N - 1 = C \\ 2 \cdot 1 - 1 = 1 \\ 4 - 1 = 3 \end{array}$$

How can we use the equation to predict other data?

a) How many circles are in Figure 71?

$$\begin{array}{l} 2(71) - 1 = C \\ 142 - 1 = C \\ 141 = C \end{array}$$

141 circles in  
Figure 71

$$2(+1) - 1 = C$$

$$142 - 1 = C$$

$$141 = C$$

141 circles in  
Figure 71

b) Which figure number has 83 circles?

$$N=? \quad C=83$$

$$2N - 1 = 83$$

$$\frac{2N}{2} = \frac{84}{2}$$

$$N = 42$$

Figure # 42 has  
83 circles.

#2) Number Patterns

25, 21, 17, 13, ... 9, 5, 1 - Term value  
1 2 3 4 5 6 7 - Term # =

a) Make a table of values for the first five terms. Use the table to create the equation for this pattern.

$d \cdot N$	Term #	Term value
-4	1	25
-8	2	21
-12	3	17
-16	4	13
-20	5	9

$$d = -4$$

$$d \cdot N + 29 = V$$

$$-4 + 29 = 25$$

$$-8 + 29 = 21$$

$$-20 + 29 = 9$$

$$\rightarrow -4N + 29 = V$$

b) Use the equation to answer the following:

i. What is the value of the 11<sup>th</sup> term?

$$-4(11) + 29 = V$$

$$-44 + 29 = -15 = \text{Value of the 11<sup>th</sup> term}$$

ii. Which term has the value of -47?

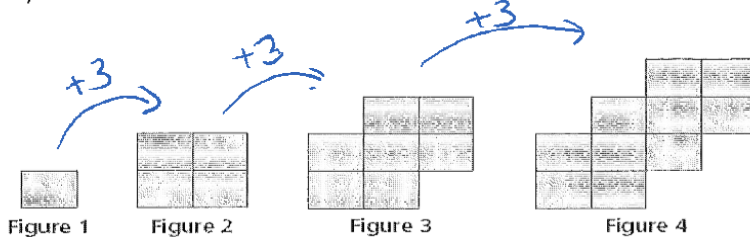
$$-4N + 29 = -47$$

$$-4N = -76$$

$$\frac{-4N}{-4} = \frac{-76}{-4}$$

N = 19<sup>th</sup> term is  
-47

#3)



① Find the difference to go from one term to the next.  $d = 3$

② Multiply the difference with the term #, N

③ relate  $d \cdot N$  to the value of the term by adding or subtracting

$$\text{Eq}^n: d \cdot N \pm \dots = V$$

④  $\text{Eq}^n: d \cdot N \pm \dots = Q$  ← # of squares

$$\text{Eq}^n: d \cdot N \pm \dots = Q$$

$$3N - 2 = Q$$

$Q \equiv \# \text{ of squares}$

$N \equiv \text{Figure \#}$

a) Describe the pattern in the figures above.

b) Create a table of values to represent the figure number and the number of squares for the first 4 figures.

$3N$	Fig #	# of squares
3	1	1
6	2	4
9	3	7
12	4	10

c) Write a linear equation to represent this pattern.

d) How many squares are in Figure 12?

$$3N - 2 = Q$$

$$3(12) - 2 = Q$$

$$34 = Q$$

e) Which figure number has 106 squares?

In figure 12, there  
are 34 squares

$$\frac{3N}{3} = \frac{6}{3}$$

e) Which figure number has 106 squares?

$$N = ?$$

$$34 = Q$$

$$Q$$

$$3N - 2 = Q$$

$$3N - 2 = 106$$

$$3N = 108$$

$$N = 36$$

Fig 36 has 106 squares.

$$\frac{3N}{3} = \frac{6}{3}$$

$$N = 2$$

Homework: p. 217 #4 (b, c, d, e), 5 & 6 (a, c, d, e), ~~7, 9, 10, 14 (a, b, c)~~

(plus the "More Practice" on pg 4) →

③

### More Practice

④

1) Determine the common difference of the following linear patterns and use it to find the next 3 numbers.

a) 5, 8, 11, 14, ....

b) -5, 1, 7, 13, ....

2) Write an equation relating C to n. (Verify it works for every pair of values)

a)

n	1	2	3	4
C	5	8	11	14

b)

n	1	2	3	4
C	-5	-11	-17	-23

3) Determine the 30<sup>th</sup> number in the following linear pattern. Write the equation first.

a) -8, -3, 2, 7...

4) The fare for a taxi ride is \$1.50 per kilometre plus a fixed cost of \$2.40.

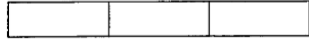
a) Write an equation for the fare  $F$  in terms of the fixed cost and the cost per  $n$  kilometres.

b) What is the fare for an 11km ride?

c) If you cost was \$32.40, how many km was the ride?

1. A banquet table seats 8 people, three on each side, and two at the ends. Tables can be

1. A banquet table seats 8 people, three on each side, and two at the ends. Tables can be connected end to end, as shown.



- a) How many additional people can be seated when a table is added?
  - b) Make a table to show how many people can sit at 1-4 tables.
  - c) Find a pattern and write an equation. Use  $n$  for the number of tables and  $P$  for the number of people.
  - d) Use your equation to determine how many people can be seated at 10 tables.
  - e) How many tables are needed to seat 344 people?
2. Joseph is considering two different payment plans for his gym membership. **Plan A** charges a flat fee of \$45.00 each month. **Plan B** charges a flat fee of \$25.00 each month plus \$2.50 per visit.
- a) Write a linear relation to represent each plan
  - b) Graph the two linear relations
  - c) Use the graph to help you identify at what month the two plans would have the same cost
  - d) If Joseph planned to visit the gym 10 times per month, which plan would cost less? How much would it cost him?