

6. 1 Representing Patterns

Prescribed Learning Outcomes (PLO'S):

- Write a linear expression representing a given pictorial, oral, or written pattern
- Write a linear equation to represent a given context
- Describe a context for a given linear equation
- Solve using a linear equation, a given problem that involves pictorial, oral, and written linear patterns
- Write a linear equation representing the pattern in a given table of values and verify the equation by substituting values from the table
- Determine, by substitution, whether a given rational number is a solution to a given linear equation

Terminology

$$\begin{array}{c} +5 = d \\ \swarrow \searrow \\ 0, 5, 10, 15 \dots \end{array}$$

Linear patterns: a sequence of numbers in which the pattern only involves addition or subtraction.

Common Difference d : is the difference between any two consecutive numbers in a linear pattern.

$$d = 5$$

Observing a Pattern

Draw the Figure 5 by continuing the pattern shown below. How is the number of circles related to the figure number? Show the relationship in a table.

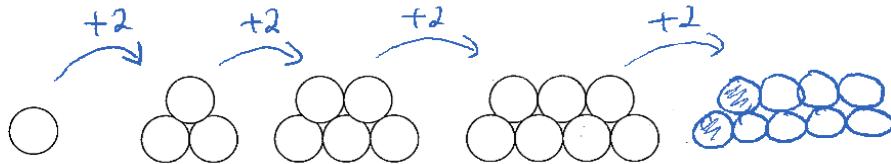


Figure 1

Figure 2

Figure 3

Figure 4

Fig 5

input	output
Figure #	# of circles
1	1
2	3
3	5
4	7
5	9

What do you notice as we move from figure to figure? What is changing? How is it changing? Look at the table. Do you see any patterns?

(Fig #)(d)	# of circles
$1 \cdot 2 = 2$	1
$2 \cdot 2 = 4$	3
$3 \cdot 2 = 6$	5
$4 \cdot 2 = 8$	7
$5 \cdot 2 = 10$	9

Make a new table of values. This time multiply the figure # by the change in the number of circles and put this on the left-hand side of your table. The right-hand side should stay the same. What can you see now?

$$d \cdot \text{input} = \text{output}$$

Write an equation relating:

the figure # (call it "N") and

the # of circles (call it "C")

$$\begin{array}{l} 2 \cdot N - 1 = C \\ 2 \cdot 1 - 1 = 1 \\ 2 \cdot 2 - 1 = 3 \end{array}$$

How can we use the equation to predict other data?

a) How many circles are in Figure 71?

$$\begin{array}{l} 2(71) - 1 = C \\ 142 - 1 = C \\ 141 = C \end{array}$$

141 circles in
Figure 71

$$\begin{aligned} 2(N+1) - 1 &= C \\ 142 - 1 &= C \\ 141 &= C \end{aligned}$$

b) Which figure number has 83 circles?

$$N = ?$$

$$C = 83$$

141 circles \rightarrow
Figure 71

$$N = 42$$

Figure # 42 has
83 circles.

#2) Number Patterns

25, 21, 17, 13, ... 9, 5, 1 - Term value
1 2 3 4 5 6 7 - Term # =

a) Make a table of values for the first five terms. Use the table to create the equation for this pattern.

$d \cdot N$	N	V
Term #	Term value	
-4	1	25
-8	2	21
-12	3	17
-16	4	13
-20	5	9

$$d = -4$$

$$\begin{aligned} d \cdot N + 29 &= V \\ -4 + 29 &= 25 \\ -8 + 29 &= 21 \\ -20 + 29 &= 9 \end{aligned}$$

$$\rightarrow -4N + 29 = V$$

b) Use the equation to answer the following:

i. What is the value of the 11th term?

$$V = ?$$

$$\begin{aligned} -4(11) + 29 &= V \\ -44 + 29 &= -15 = \text{Value of the 11th term} \end{aligned}$$

ii. Which term has the value of -47?

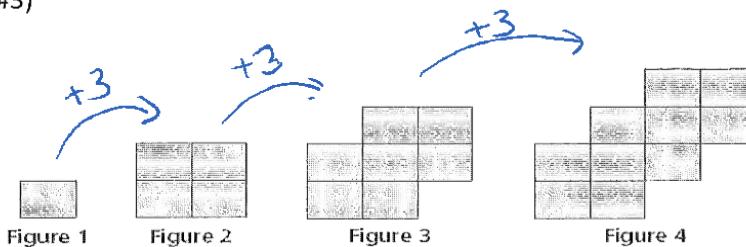
$$N = ?$$

$$\begin{aligned} -4N + 29 &= -47 \\ -29 &= -47 \\ -4N &= -76 \end{aligned}$$

$$\rightarrow -\frac{4N}{-4} = \frac{-76}{-4}$$

$N = 19^{\text{th}}$ term is
-47

#3)



- Find the difference to go from one term to the next. $d =$ $+3$
- Multiply the difference with the term #, N
- Relate $d \cdot N$ to the value of the term by adding or subtracting
- Eqn: $d \cdot N + \text{# of squares} = V$

- Describe the pattern in the figures above.
- Create a table of values to represent the figure number and the number of squares for the first 4 figures.
- Write a linear equation to represent this pattern.

N	Fig #	# of squares
3	1	1
6	2	4
9	3	7
12	4	10

$$\begin{aligned} \text{Eqn: } dN + \text{# of squares} &= V \\ 3N - 2 &= Q \end{aligned}$$

$Q = \text{# of squares}$
 $N = \text{Figure #}$

d) How many squares are in Figure 12?

$$\begin{aligned} 3N - 2 &= Q \\ 3(12) - 2 &= Q \\ 34 &= Q \end{aligned}$$

e) Which figure number has 106 squares?

$$\begin{aligned} 3N - 2 &= 106 \\ 3N &= 108 \\ N &= 36 \end{aligned}$$

In figure 12, there
are 34 squares

$$3N = \frac{6}{2}$$

e) Which figure number has 106 squares?

$$N=?$$

$$3N - 2 = Q$$

$$Q$$

$$3N - 2 = Q$$

$$3N - 2 = 106 + 2$$

$$3N = \frac{108}{3}$$

$$N = 36$$

Homework: p. 217 #4 (b, c, d, e), 5 & 6 (a, c, d, e), 7, 9, 10, 14 (a, b, c)

Fig 36 has 106 squares.

$$\frac{3N - 2}{3} = 6 \\ N = 2$$

(plus the "More Practice" on pg 4) →

③

④ More Practice

④

1) Determine the common difference of the following linear patterns and use it to find the next 3 numbers.

a) 5, 8, 11, 14, ...

b) -5, 1, 7, 13, ...

2) Write an equation relating C to n. (Verify it works for every pair of values)

a)
$$\begin{array}{c|cccc} n & 1 & 2 & 3 & 4 \\ \hline C & 5 & 8 & 11 & 14 \end{array}$$

b)
$$\begin{array}{c|cccc} n & 1 & 2 & 3 & 4 \\ \hline C & -5 & -11 & -17 & -23 \end{array}$$

3) Determine the 30th number in the following linear pattern. Write the equation first.

a) -8, -3, 2, 7...

4) The fare for a taxi ride is \$1.50 per kilometre plus a fixed cost of \$2.40.

a) Write an equation for the fare F in terms of the fixed cost and the cost per n kilometres.

b) What is the fare for an 11km ride?

c) If you cost was \$32.40, how many km was the ride?

1. A banquet table seats 8 people, three on each side, and two at the ends. Tables can be

1. A banquet table seats 8 people, three on each side, and two at the ends. Tables can be connected end to end, as shown.



- a) How many additional people can be seated when a table is added?
- b) Make a table to show how many people can sit at 1-4 tables.
- c) Find a pattern and write an equation. Use n for the number of tables and P for the number of people.
- d) Use your equation to determine how many people can be seated at 10 tables.
- e) How many tables are needed to seat 344 people?

2. Joseph is considering two different payment plans for his gym membership. **Plan A** charges a flat fee of \$45.00 each month. **Plan B** charges a flat fee of \$25.00 each month plus \$2.50 per visit.

- a) Write a linear relation to represent each plan
- b) Graph the two linear relations
- c) Use the graph to help you identify at what month the two plans would have the same cost
- d) If Joseph planned to visit the gym 10 times per month, which plan would cost less? How much would it cost him?