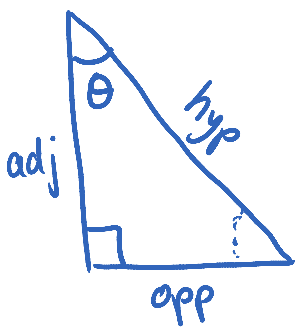
Foundations 11 | Unit 3: lesson 1

Trig Ratios of Obtuse Angles [4.1]

Warm Up:

- Write down the Primary Trig Ratios
- Draw the type of triangle that they apply to
- Identify the sides of the triangle referred to in the ratios



$$\sin\theta = \frac{opp}{hyp}$$

$$\cos\theta = \frac{adj}{hyp}$$

$$\tan\theta = \frac{opp}{adj}$$

⇨ What does "sin 80° = 0.9848…." mean? the ratio/fraction of 2 sides

In small groups, complete the table provided on the back of this page.

Here are some hints to guide you:

- Do you see similarities when you look at the sine ratios or just the cosine ratios?
- Do you see any patterns when you look at the ratios for the supplementary angles?

Before you go, here's what you need to know:

The sine ratio for supplementary angles are equal

$$\sin\theta = \sin(180° - \theta)$$

The cosine and tangent ratios for supplementary angles are opposite signs.

$$\cos\theta = -\cos(180° - \theta)$$

$$\tan\theta = -\tan(180° - \theta)$$

ex $\cos 70° = -\cos 110°$

$-\cos 70° = \cos 110°$

Practice 163 , 4

Practice
pg 163
#1-4

ex $\cos 70 = -\cos 110$
$-\cos 70^\circ = \cos 110^\circ$

Supplementary Angle Ratio Patterns

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$(180^\circ - \theta)$	$\sin(180^\circ - \theta)$	$\cos(180^\circ - \theta)$	$\tan(180^\circ - \theta)$
100°	0.9848	-0.1736	-5.6713	80°	sin 80° 0.9848	cos 80 0.1736	5.6713
110°	0.9397	-0.3420	-2.7475	70°	0.9397	0.3420	2.7475
120°	0.8660	-0.5000	-1.7321	60°	0.8660	0.5000	1.7321
130°							
140°							
150°							
160°							
170°							
180°							

θ	100	110	120	130	140	150	160	170	180

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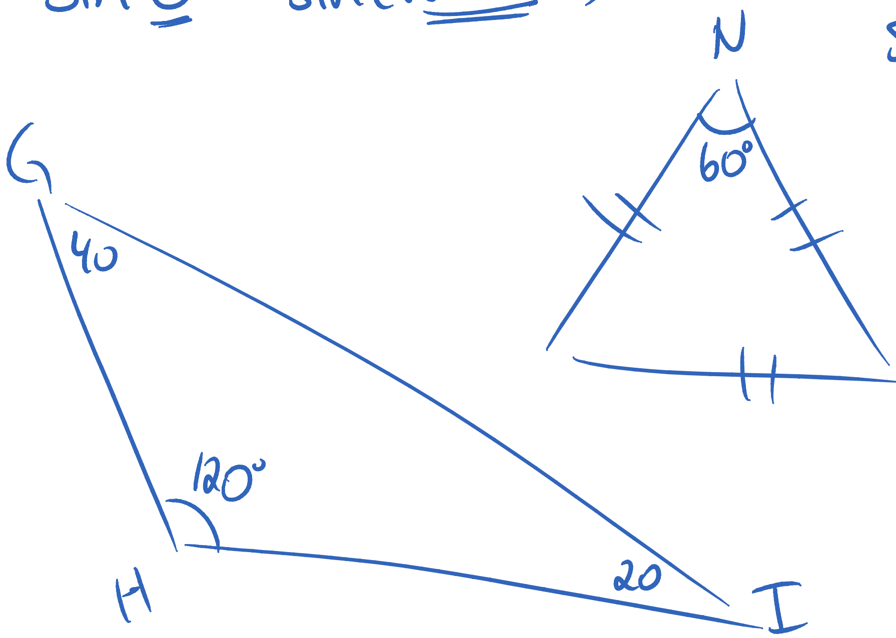
4) $\sin\theta = \sin(180-\theta)$



180-60

$\sin 60 = \sin 120$

$-\cos 60 = \cos 120$

$-\tan 60 = \tan 120$

3) a) $\sin\theta = 0.64$ ← ratio of sides $\frac{opp}{hyp}$

$\sin^{-1} 0.64 = \theta = 40°$

$180 - 40° = 140°$

$\sin 40° = \sin 140° = 0.64$

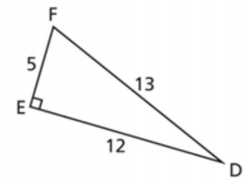
4.1 Homework

Name: ______________________________ Date: ______________________

Trigonometry Review

For questions 1 and 2, choose the correct answer: A, B, C, or D

1. For ΔDEF, how many of these statements are true? cos



$\angle D = \frac{12}{13}$ $\sin \angle D = \frac{5}{13}$ tan

$\angle D = \frac{5}{12}$ $\tan \angle F = 2.4$

A. 1 is true. **B.** 2 are true. **C.** 3 are true. **D.** All are true.

2. In right ΔDEF, with $\angle E = 90°$, which statement is false?

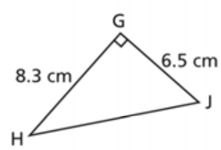
As $\angle D$ decreases:

A. $\sin \angle D$ increases. **B.** $\sin \angle F$ increases.

C. $\cos \angle D$ increases. **D.** $\cos \angle F$ decreases.

3. a) Solve each triangle. Give your answers to the nearest tenth.

i)



ii) Right ΔKMN with $\angle M = 90°$, $\angle N = 26°$, and KN = 15.0 cm.

b) When you solved the triangles in part a, did you use the same strategies?
If your answer is yes, describe your strategy.
If your answer is no, explain why you used different strategies.

4. The angle of inclination of a conveyor is 8°. The conveyor rises 0.75 m.
What is the length of the conveyor? Give your answer to the nearest hundredth of a metre.

5. A helicopter is hovering at a height of 300 m.
From the helicopter, the angle of depression of the top of a wind turbine is 40°
and the angle of depression of the base of the turbine is 48°.
Determine the height of the turbine, to the nearest tenth of a metre.

ANSWERS TO Master 2.7 Chapter Test

1. D

2. A

3. a) i) JH ≐ 10.5 cm; ∠H ≐ 38.1°; ∠J ≐ 51.9°

ii) MN ≐ 13.5 cm; KM ≐ 6.6 cm; ∠K = 64°

b) Answers may vary. No, I used different strategies. In ΔGHJ, I used the Pythagorean Theorem first to calculate the length of the third side, then I used the tangent ratio to calculate the angle measures. In ΔKMN, I used the sine and cosine ratios to calculate the lengths of the legs, then I subtracted the given angle from 90° to calculate the other acute angle. I didn't need the Pythagorean Theorem.

4. The conveyor is about 5.39 m long.

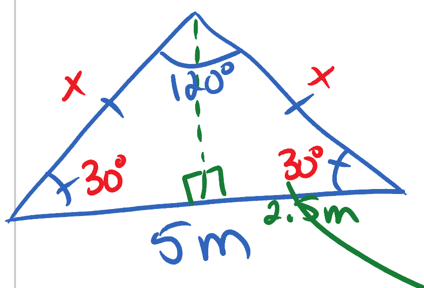
5. TB = 73.3 m

Sine & Cosine Law for Obtuse Angles – Part 1 [4.2]

one angle in Δ is bigger than 90°

Explore: An isosceles obtuse triangle has one angle that measures 120° and one side length that is 5m. What could the other side lengths be?

(Think about: What do you know about Isos Δs? How many Δs could you draw? What happens if you look at the height of the Δ and then the relationship with the 2 smaller Δs?)



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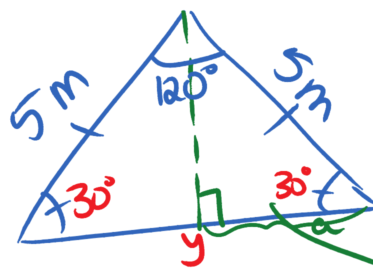


$\frac{5m}{\sin 120^\circ} = \frac{x}{\sin 30^\circ}$

$x = 2.9m$

$\cos 30^\circ = \frac{2.5}{x}$

$x = 2.9m$

$\frac{5m}{\sin 30^\circ} = \frac{y}{\sin 120^\circ}$

$y = 8.7m$

$\cos 30^\circ = \frac{a}{5m}$

$a = 4.33$

$2a = 8.7m$

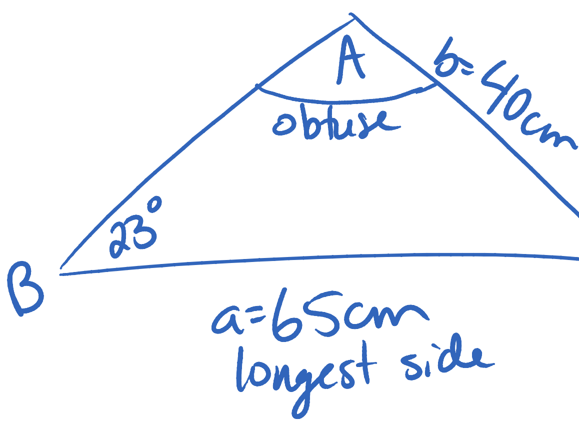
A Few Notes ...

acute or obtuse?

- Be careful when using the sine law to determine the measure of an angle. The inverse sine of a ratio always gives an acute angle, but the supplementary angle has the same ratio. You must decide whether the acute angle, θ, or the obtuse angle, $180° - \theta$, is the correct angle for your triangle.
- Because the cosine ratios for an angle and its supplement are not equal (they are opposites), the measures of the angles determined using the cosine law are always correct.

Example 1:

In an obtuse triangle, ∠ B measures 23.0° and its opposite side, *b*, has a length of 40.0 cm. Side *a* is the longest side of the triangle, with a length of 65.0 cm. Determine the measure of ∠ A to the nearest tenth of a degree.



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{65} = \frac{\sin 23}{40}$$

$$\sin A = 0.6349$$

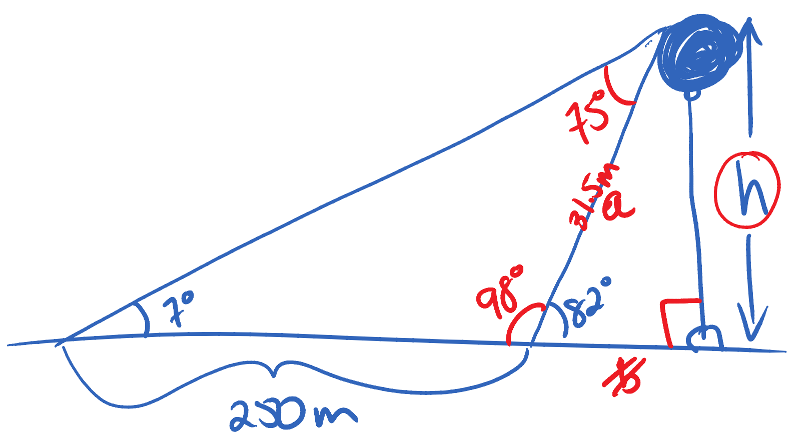
$$A = \sin^{-1} 0.6349$$

$$A = 39.4° \quad \text{obtuse? No}$$

$$A = 180 - 39.4° = 140.6° \checkmark$$

Example 2:

Colleen and Juan observed a tethered balloon advertising the opening of a new fitness centre. They were 250m apart, joined by a line that passed directly below the balloon, and were on the same side of the balloon. Juan observed the balloon at an angle of elevation of 7° while Colleen observed the balloon at an angle of elevation of 82°. Determine the height of the balloon to the nearest metre.



$$\frac{a}{\sin 7°} = \frac{250}{\sin 75°}$$

$$a = 31.5m$$

height

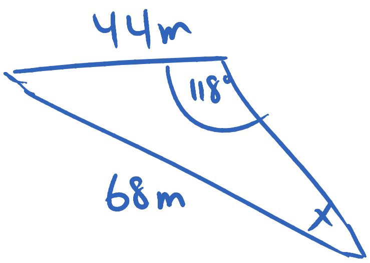
$$\sin 82° = \frac{h}{31.5m}$$

$$h = 31.5(\sin 82)$$

$$= 31m$$

Practice pg 171 # (4,5) ac, 6, 8, 13, 17

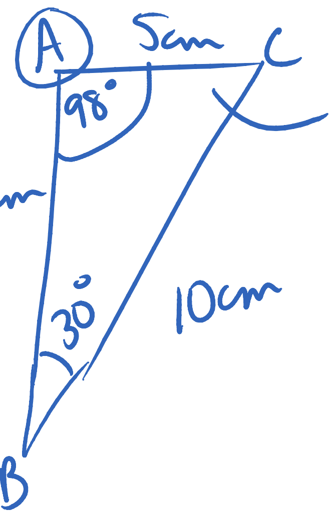
4a)



$$\frac{\sin x}{44} = \frac{\sin 118}{68}$$

$x = 35°$ ✓

5c)



$180 - 98 - 30$
$= 52°$

cosine law (A)

$10^2 = 5^2 + 8^2 - 2(5)(8)\cos A$

$100 - 89 = -80\cos A$

$\frac{11}{-80} = \cos A$

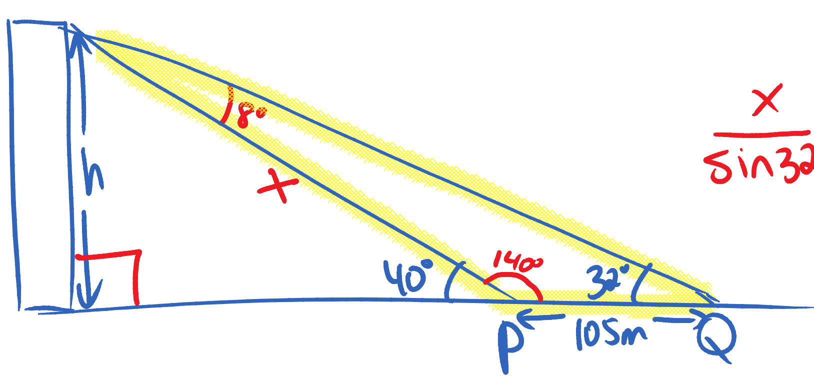
$98° = A$

Sine law (B)

$$\frac{\sin B}{5cm} = \frac{\sin 98}{10}$$

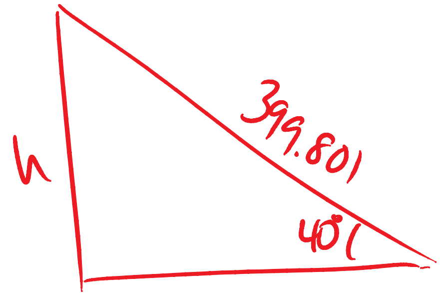
$B = 30°$

(13)



$$\frac{x}{\sin 32°} = \frac{105}{\sin 8°}$$

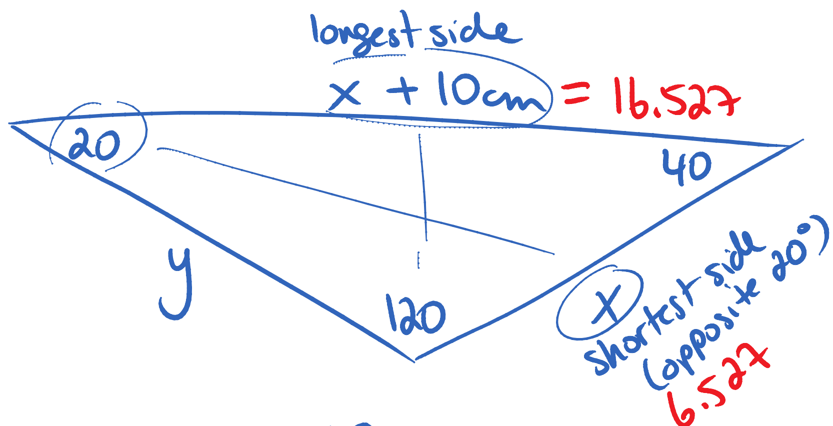
$x = 399.801\text{m}$



$$\sin 40° = \frac{h}{399.801}$$

$$h = 257.0\text{m}$$

17



1

$$\frac{x}{\sin 20°} = \frac{x+10}{\sin 120°}$$

$$(\sin 120)x = (\sin 20)(x+10)$$

$$(\sin 120)(x) - (\sin 20)(x) = (\sin 20)(10)$$

$$\frac{0.524005\,x}{0.524005} = \frac{(\sin 20)(10)}{0.524005}$$

$$x = 6.527$$

2

$$\frac{y}{\sin 40°} = \frac{6.527}{\sin 20°}$$

$$y = 12.2668$$

3 Perimeter

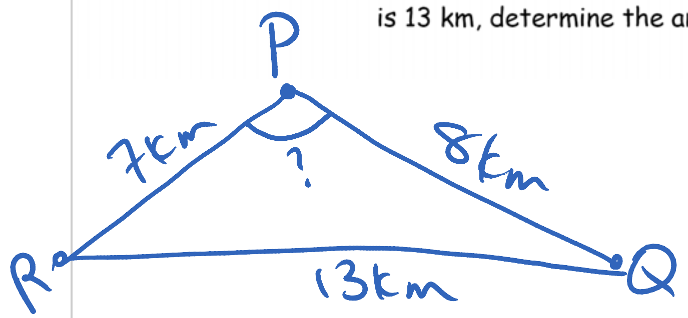
$$6.527 + 16.527 + 12.2668$$

$$= 35\text{cm}$$

4.2 (2) Sine & Cosine Law for Obtuse

Sine & Cosine Law for Obtuse Angles – Part 2 [4.2]

Example 1: Two ships set sail from, P, heading in different directions. The first ship sails 7 km to R and the second ship sails 8 km to Q. If the distance between R and Q is 13 km, determine the angle between the directions of the two ships.



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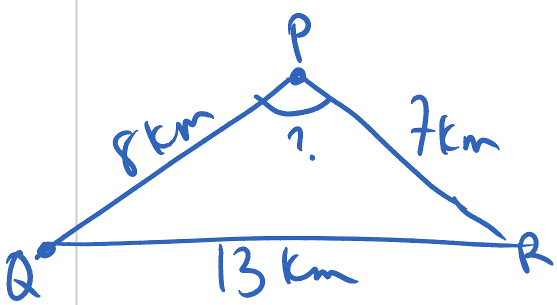


$$13^2 = 7^2 + 8^2 - 2(7)(8)\cos P$$

$$13^2 - 7^2 - 8^2 = -2(56)\cos P$$

$$\frac{56}{-112} = \frac{\cancel{-112}\cos P}{\cancel{-112}}$$

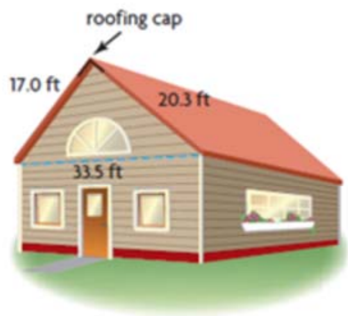
$$\cos P = -0.5$$

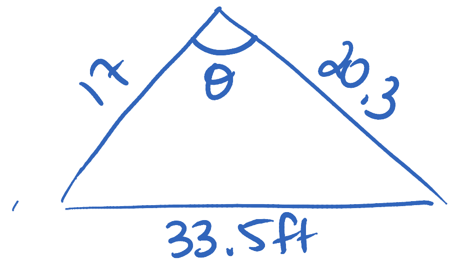
neg cosine ratio → obtuse angle

$$P = \cos^{-1}(-0.5)$$

$$= 120^\circ$$

Example 2: The roof of a house consists of two slanted sections, as shown. A roofing cap is being made to fit the crown of the roof, where the two slanted sections meet. Determine the measure of the angle needed for the roofing cap, to the nearest tenth of a degree.





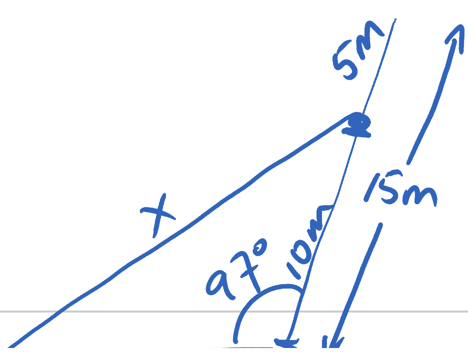
$$33.5^2 = 17^2 + 20.3^2 - 2(17)(20.3)\cos\theta$$

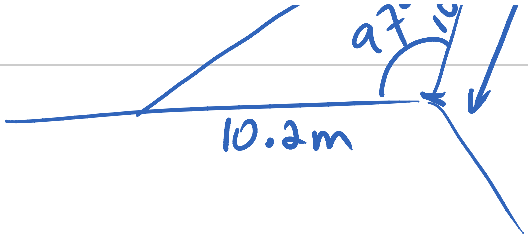
$$\frac{421.16}{-690.2} = \frac{\cancel{-690.2}\cos\theta}{\cancel{-690.2}}$$

$$\theta = 127.6^\circ$$

Practice pg 171 # 2, 7, 12, 14

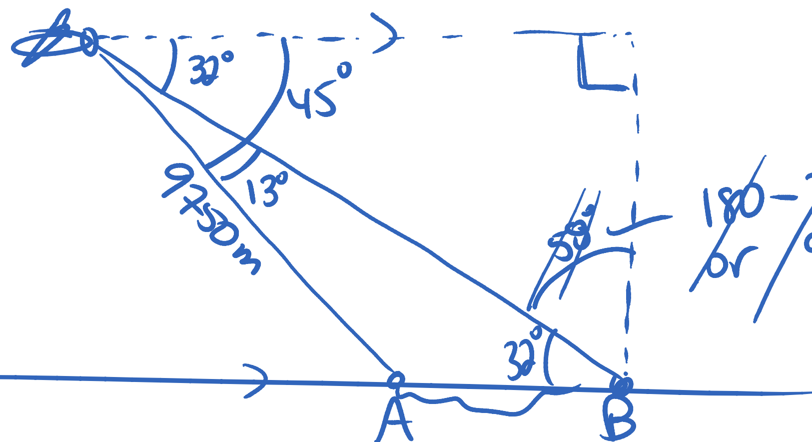
12





$$x^2 = 10.2^2 + 10^2 - 2(10.2)(10)\cos 97^\circ$$

(14)



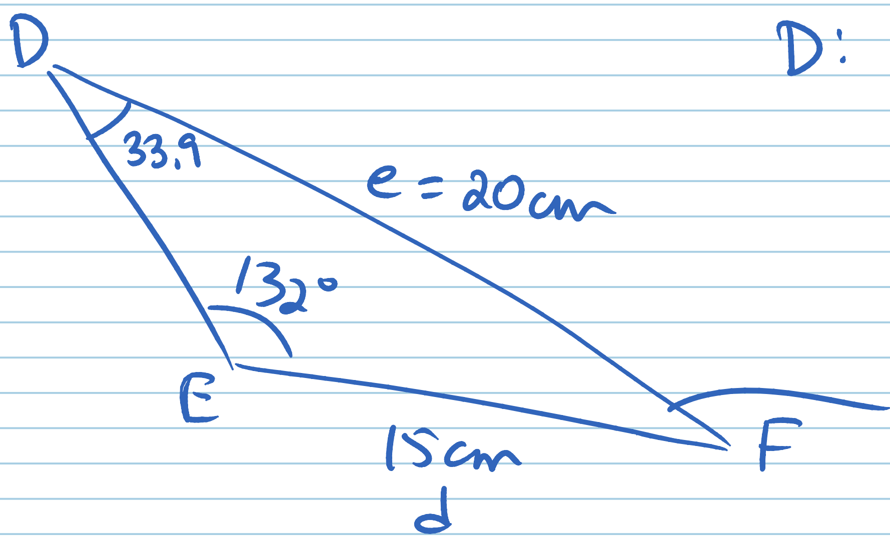
~~180 − 32° − 90° or 90° − 32° = 58°~~

$$\frac{AB}{\sin 13^\circ} = \frac{9750\text{m}}{\sin 32^\circ} \qquad \frac{\sin 13}{AB} = \frac{\sin 32}{9750}$$

$$AB = 4139\text{m}$$

4.2 ~~Homework~~ Practice pg 175 # 1, 3, 4, 5, 6a, 8, 9

⑤

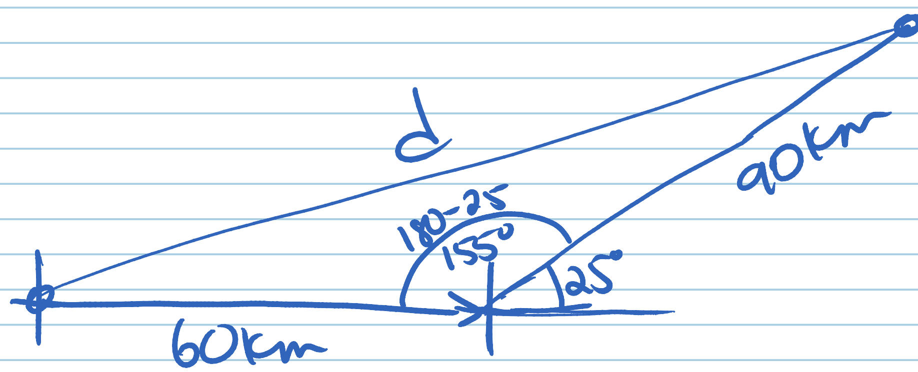


D: $\frac{\sin D}{15} = \frac{\sin 132}{20}$

D = 33.9

180 − 33.9 − 132
= 14°

⑨



$d^2 = 60^2 + 90^2 - 2(60)(90)\cos 155$

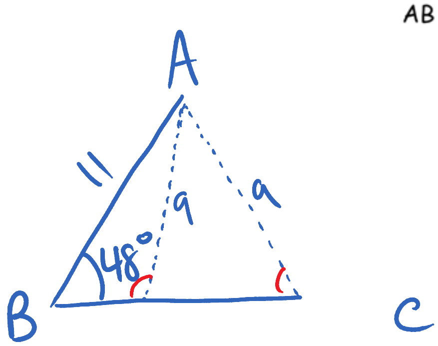
Sine Law: The Ambiguous Case – Part 1 [4.3]

When given two sides and an angle other than the contained angle, there are two possible triangles.

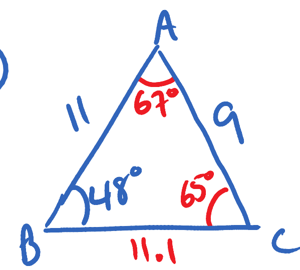
Draw two non-right triangles that fit the following criteria, and solve each triangle.

find all missing sides and angles

AB = 11 $\angle$ B = 48° AC = 9



Δ1



$$\frac{\sin C}{11} = \frac{\sin 48}{9}$$

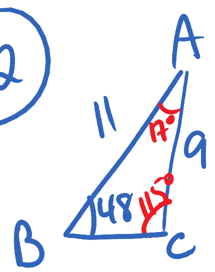
$C = 65°$

$\angle A = 180 - 48 - 65 = 67°$

$$\frac{BC}{\sin 67} = \frac{9}{\sin 48}$$

$BC = \underline{\underline{11.1}}$

Δ2



$$\frac{\sin C}{11} = \frac{\sin 48}{9}$$

$C = 65°$ acute

need obtuse $180 - 65 = 115°$

$\angle A = 180 - 115 - 48 = 17°$

$$\frac{BC}{\sin 17} = \frac{9}{\sin 48} \rightarrow BC = \underline{\underline{3.5}}$$

If Δ ABC is known to exist, and the measure of $\angle$ B, the length of side b and side c are given…

Exactly two different triangles satisfy these conditions provided **c(Sin B) < b < c**.

Otherwise there is exactly one triangle.

side-side-angle

Given each *SSA* situation for $\triangle ABC$, determine how many triangles are possible.

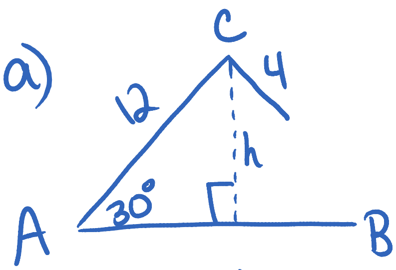
a) $\angle A = 30°$, $a = 4$ m, and $b = 12$ m

b) $\angle A = 30°$, $a = 6$ m, and $b = 12$ m

c) $\angle A = 30°$, $a = 8$ m, and $b = 12$ m

d) $\angle A = 30°$, $a = 15$ m, and $b = 12$ m

a)



$a = 4 \quad h = 6$

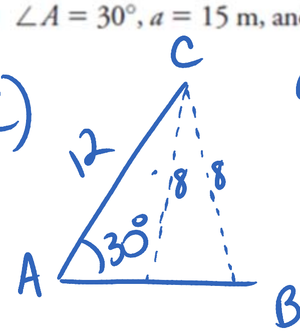
$\boxed{a < h \text{ No Triangle}}$

$\sin 30° = \frac{h}{12}$

$h = 12 \sin 30°$

$= 6m$

c)

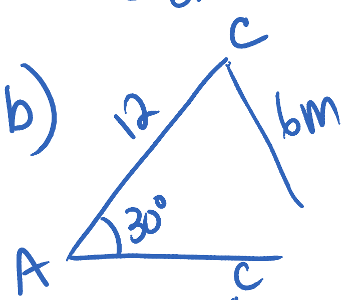


$a = 8 \quad h = 6 \quad b = 12$

$\boxed{a > h \quad a < b \text{ Two Triangles}}$

obtuse + acute

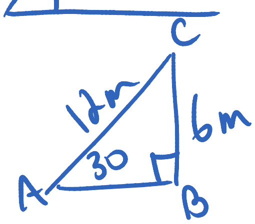
b)



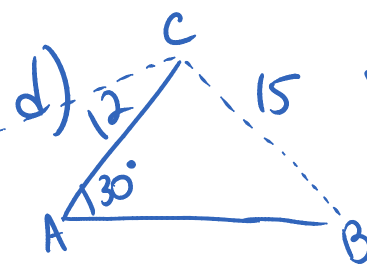
$a = 6 \quad h = 6$

$\boxed{a = h \text{ One Triangle}}$

right angle



d)



$a = 15 \quad b = 12$

$\boxed{a > b \text{ One Triangle}}$

In Summary

Key Idea

- The ambiguous case of the sine law may occur when you are given two side lengths and the measure of an angle that is opposite one of these sides. Depending on the measure of the given angle and the lengths of the given sides, you may need to construct and solve zero, one, or two triangles.

Step 0: Draw the situation

Step 1: is the side length across from the given angle longer than the other given side?

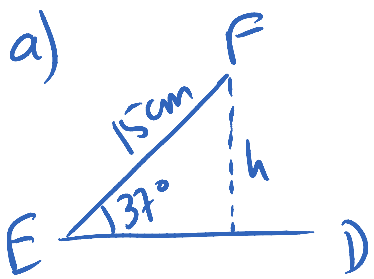
if yes → 1 △

if yes → 1 Δ
if no → step 2

Step 2: calculate the height of the Δ

Practice pg 183 # 4, 5, 6

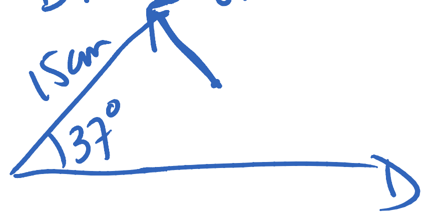
5 a)



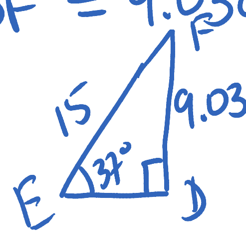
$\sin 37 = \frac{h}{15}$

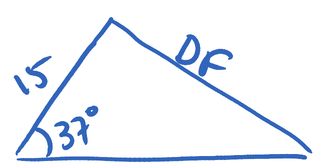
$h = 9.03\text{cm}$

b) zero Δ's : DF is shorter than 9.03cm

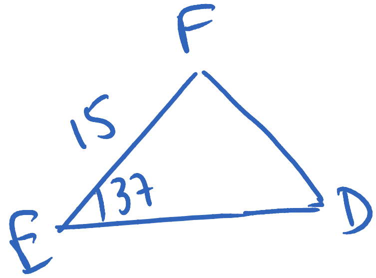


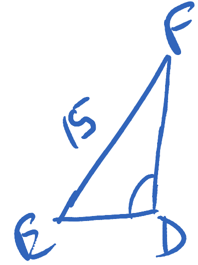
one Δ : DF = 9.03cm | DF > 15cm





two Δ's : DF > 9.03cm DF < 15cm





acute obtuse

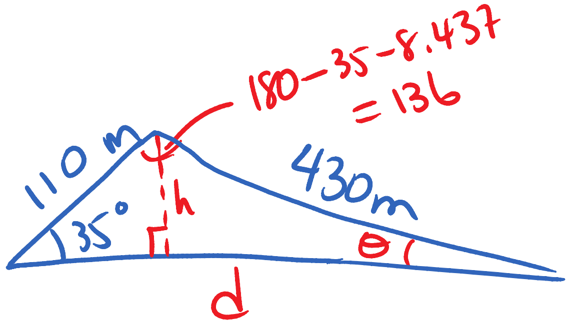
⑥

$\sin 35 = \frac{h}{430}$

$h = 246.6\text{m}$

110m is too short to make this △.

or



$\sin 35 = \frac{h}{110}$

$h = 63.1\text{ m}$

$\left.\begin{matrix} 430 > 63.1 \\ 430 > 110 \end{matrix}\right\}$ 1 △ possible

$\frac{\sin\theta}{110} = \frac{\sin 35}{430}$

$\theta = 8.437°$

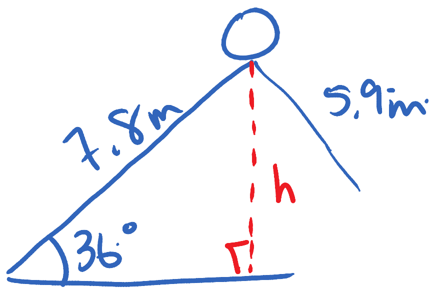
$d^2 = 110^2 + 430^2 - 2(110)(430)\cos 136$

$d = 515\text{ m}$

4.3 (2) The Ambiguous Case

Sine Law: The Ambiguous Case – Part 2 [4.3]

Martina & Carl are part of a team that is studying weather patterns. The team is about to launch a weather balloon to collect data. Martina's rope is 7.8 m long and makes an angle of 36.0° with the ground. Carl's rope is 5.9 m long. Assuming that Martina & Carl form a triangle in a vertical plane with the weather balloon, what is the distance between Martina & Carl, to the nearest tenth of a metre?

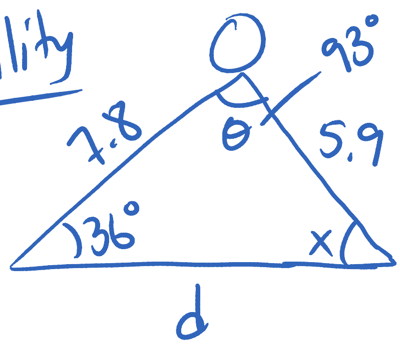


$h = 7.8 \sin 36^\circ$
$= 4.5847\text{m}$

$5.9\text{m} > h \qquad 5.9 < 7.8$

$\therefore$ 2 Δ's

1 Possibility



$$\frac{\sin x}{7.8} = \frac{\sin 36^\circ}{5.9}$$

$x = 51^\circ$

$\theta = 180 - 51 - 36 = 93^\circ$

$$\frac{d}{\sin 93^\circ} = \frac{5.9}{\sin 36^\circ}$$

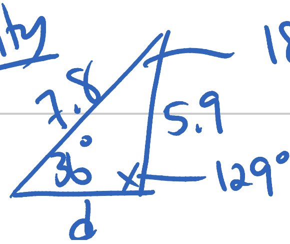
$d = 10\text{m}$

$$\frac{\sin x}{7.8} = \frac{\sin 36}{5.9}$$

$x = 51^\circ$
but obtuse

$180 - 51 = 129^\circ$

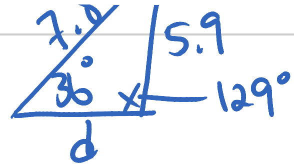
2 Possibility



$180 - 129 - 36$
$= 15^\circ$

$$\frac{d}{\sin 15^\circ} = \frac{5.9}{\sin 36}$$

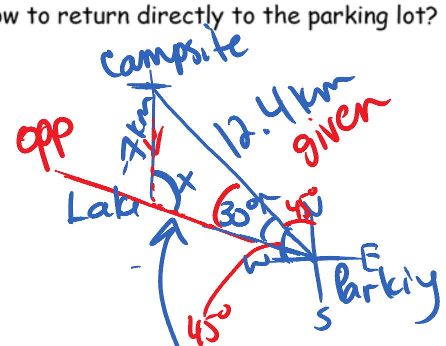
$d = 2.6\text{m}$



$= 15$

$$\frac{d}{\sin 15^\circ} = \frac{5.9}{\sin 36} \qquad d = 2.6m$$

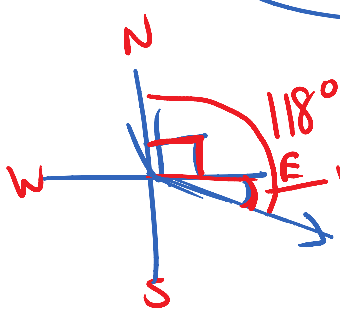
Leanne & Kerry are hiking in the mountains. They left Leanne's car in the parking lot and walked northwest for 12.4 km to a campsite. Then they turned due south and walked another 7.0 km to a glacier lake. The weather was taking a turn for the worse, so they decided to plot a course directly back to the parking lot. Kerry remembered, from the map in the parking lot, that the angle between the path to the campsite and the path to the glacier lake measures about 30°. What compass direction should they follow to return directly to the parking lot?



$$\frac{\sin x}{12.4} = \frac{\sin 30}{7}$$

$x = 62°$ not obtuse

$x = 180 - 62 = 118°$



$118 - 90 = 28°$ ⟹ head 28° S of E

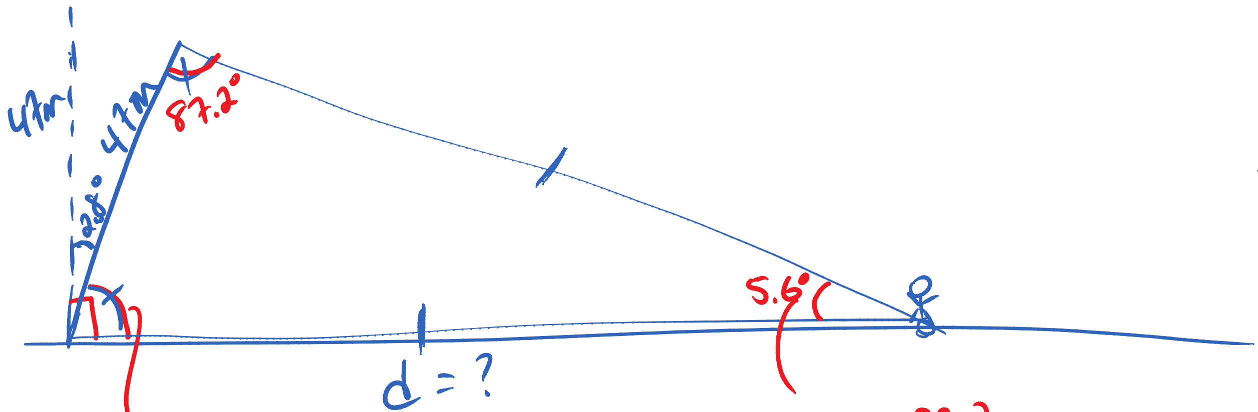
E 28° S

Practice pg 183 #10, 13, 17

Before you go

Explain what is meant by the ambiguous case of the sine law. Describe situations in which a sine law problem may have no solution, one solution or two solutions.

(13)

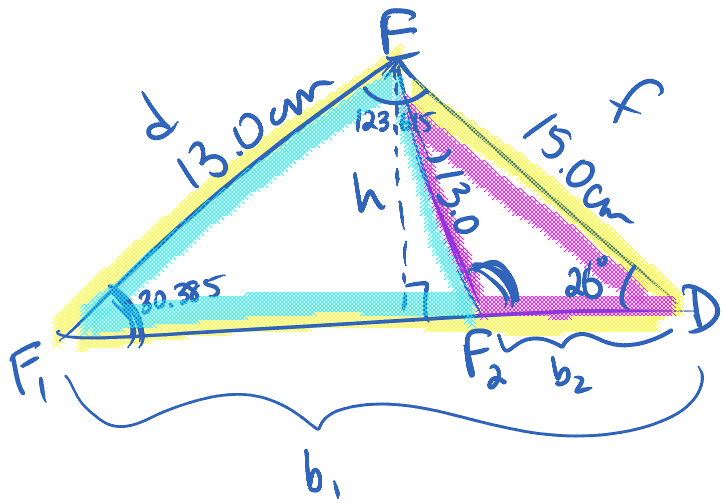


$90° - 2.8° = 87.2°$

$180 - 87.2 - 87.2 = 5.6°$

$$\frac{d}{\sin 87.2°} = \frac{47}{\sin 5.6°} \qquad d = 481\text{m}$$

(17)



$$\text{Area}_{DEF_1} = \frac{bh}{2}$$

$h = 15\text{cm} \sin 26°$

$h = 6.576\text{cm}$

b_1: $\dfrac{\sin F_1}{15} = \dfrac{\sin 26}{13}$

$F_1 = 30.385°$ acute ✓

$\angle E = 180 - 26 - 30.385$
$= 123.615$

$$\frac{b_1}{\sin 123.615} = \frac{13}{\sin 26°}$$

$b_1 = 24.696$

$$A_{DEF_1} = \frac{b_1 h}{2} = \frac{(24.696)(6.576)}{2} = 81.2\text{cm}^2$$

---

b) $A_{DEF_2} = \dfrac{b_2 h}{2}$

b_2: $\dfrac{\sin F_2}{15} = \dfrac{\sin 26}{12}$

$= \frac{2.267 cm (6.576 cm)}{2}$

$= 7.5 cm^2$

c) $A_{DEF_1} - A_{DEF_2} = A_{F_1EF_2}$

13

$F_2 = 30.385$

obtuse?

$180 - 30.385 = 149.615$

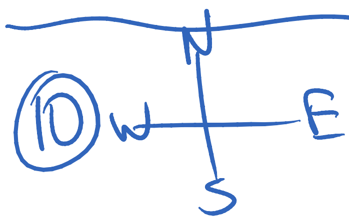
$\angle E = 180 - 149.615 - 26$

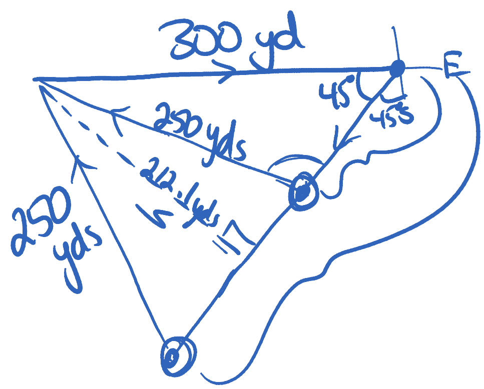
$= 4.385$

$\frac{b_2}{\sin 4.385} = \frac{13}{\sin 26}$

$b_2 = 2.267$

10



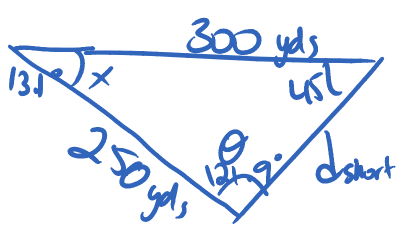


$h = 300 \sin 45°$

$= 212.1$ yds

$250 > h \therefore 2$ triangles

shorter distance



$\left\{ \frac{\sin\theta}{300} = \frac{\sin 45}{250} \right\}$

$\theta = 58.1°$

acute

s.b. obtuse

$180 - 58.1 = 121.9°$

$x = 180 - 121.9 - 45$

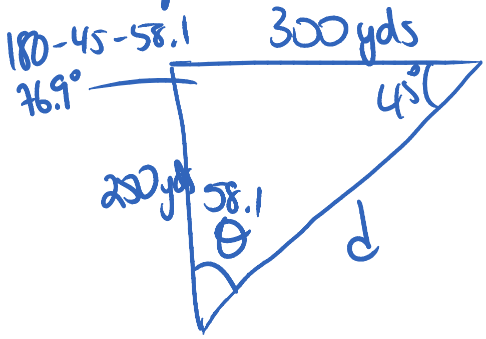
$= 13.1°$

$\frac{d_{short}}{\sin 13.1°} = \frac{250}{\sin 45°}$

$d_{short} = 80$ yds

or

longer distance



$$\frac{\sin\theta}{300} = \frac{\sin 45}{250}$$

$\theta = 58.1$
acute ✓

$$d^2 = 250^2 + 300^2 - 2(250)(300)\cos 76.9°$$

$d = 344$ yds

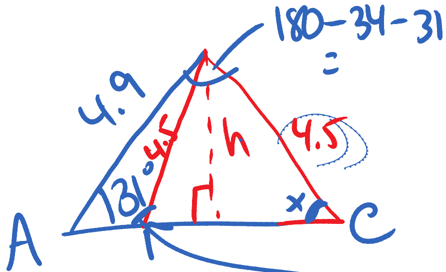
4.4 Solving Problems using Obtuse Triangles

Solving Problems using Obtuse Triangles [4.4]

Quick Review:

Find all possible measures of $\angle C$ in the following triangles:

a) ΔABC where $\angle A = 31°$, $a = 4.5$ cm and $c = 4.9$ cm



$\sin 31° = \frac{h}{4.9}$

$h = 4.9 \sin 31°$

$= 2.5$

$2.5 < 4.5 \therefore 2$ triangles

Big Δ

$\frac{\sin x}{4.9} = \frac{\sin 31}{4.5}$

$x = 34° = \angle C$

acute

$\angle C = 180 - 34° = 146°$

obtuse

b) ΔABC where $\angle A = 61°$, $a = 7.5$ cm, and $c = 5.8$ cm

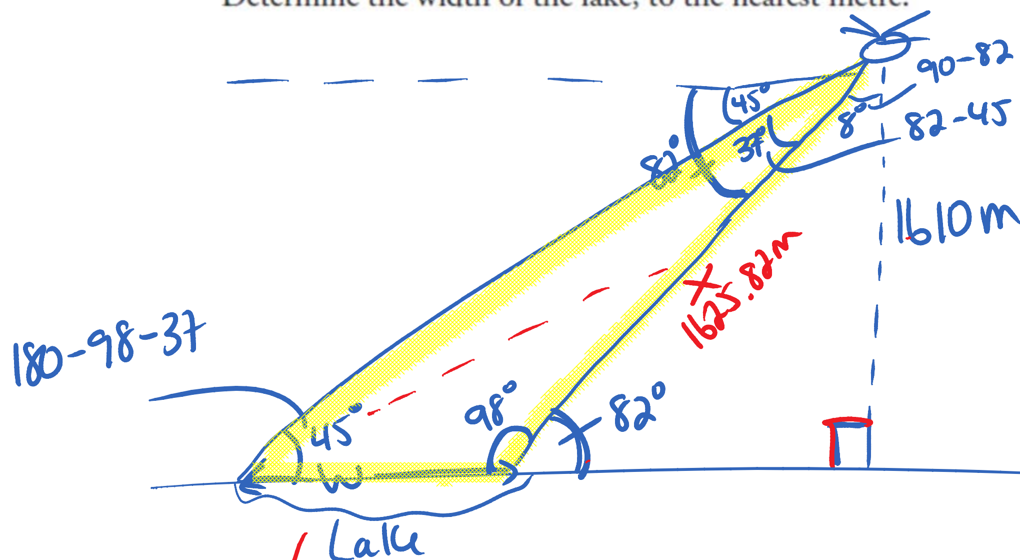
$7.5 > 5.8 \therefore 1\ \Delta$

$\frac{\sin C}{5.8} = \frac{\sin 61°}{7.5}$

$C = 43°$

A surveyor in a helicopter would like to know the width of Garibaldi Lake in British Columbia. When the helicopter is hovering at 1610 m above the forest, the surveyor observes that the angles of depression to two points on opposite shores of the lake measure 45° and 82°. The helicopter and the two points are in the same vertical plane.

Determine the width of the lake, to the nearest metre.



$\sin 82 = \frac{1610m}{x}$

$x = \frac{1610m}{\sin 82}$

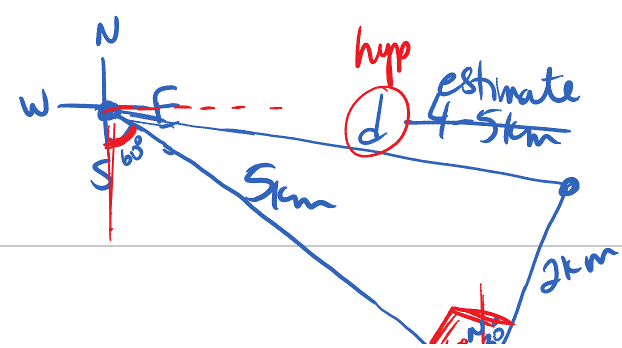
$x = 1625.82m$

$\frac{w}{\sin 37°} = \frac{1625.82}{\sin 45°}$

$w = 1384m$

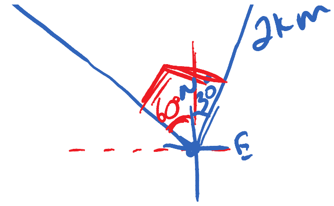
Practice pg 194 # 4–6, 10, 13

5



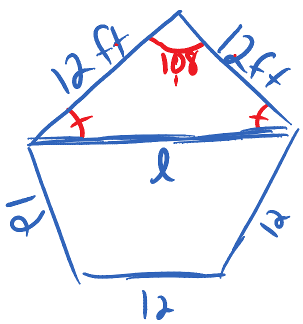
$d^2 = 5^2 + 2^2$

$$d^2 = 5^2 + 2^2$$
$$= 25 + 4$$
$$d = \sqrt{29} = 5.4\text{ km}$$

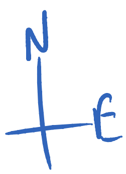


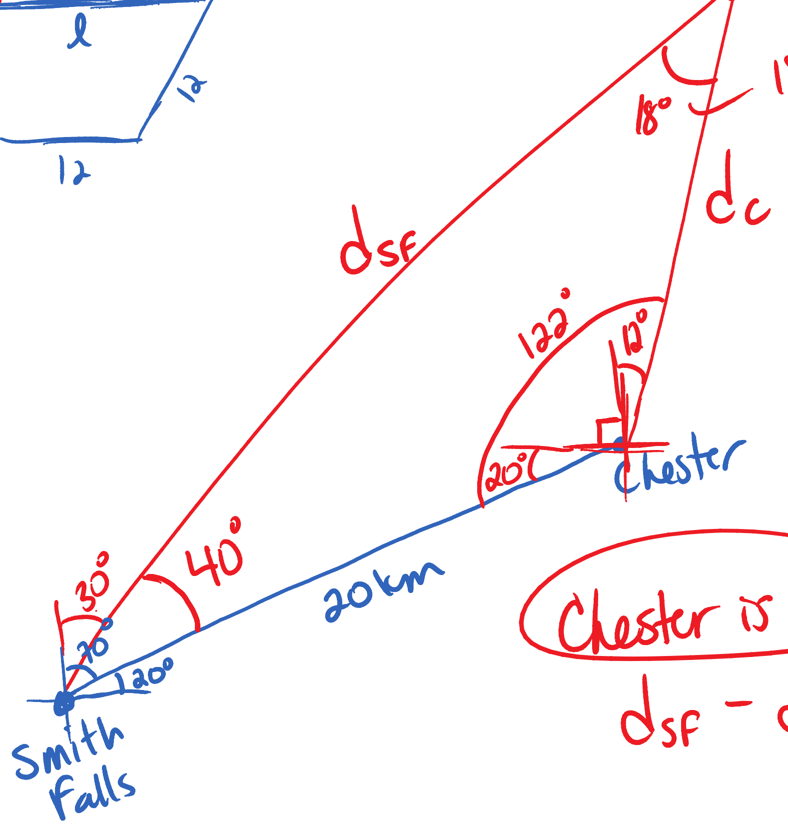
(4)

$$\text{int} \angle = \frac{(\overset{n}{5}-2)180°}{} = 108°$$



(10)





$180 - 122 - 40$
$= 18$

Chester is closer

$d_{SF} - d_c$ = how much closer

$$\frac{d_{SF}}{\sin 122°} = \frac{20\text{km}}{\sin 18°}$$
$$d_{SF} = 54.89\text{ km}$$

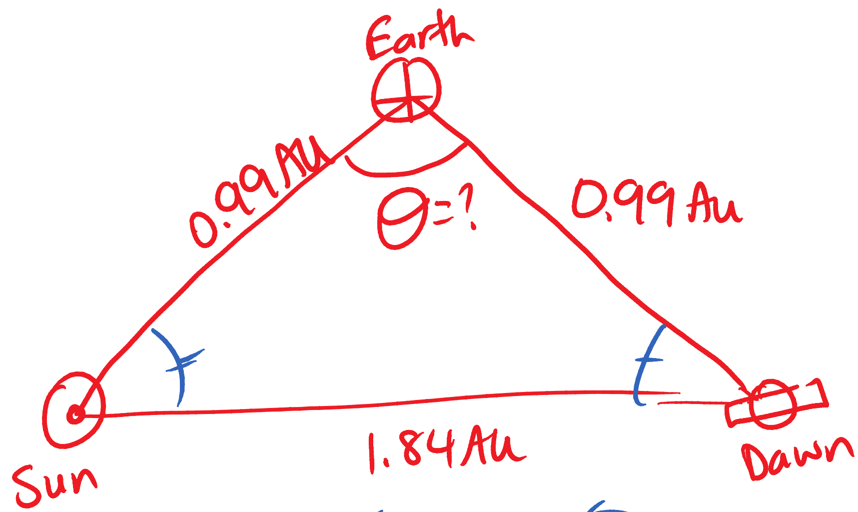
$$\frac{d_c}{\sin 40°} = \frac{20\text{km}}{\sin 18°}$$
$$d_c = 41.60\text{ km}$$

$$54.89 - 41.60 = 13\text{ km}$$

(1)

Earth

6

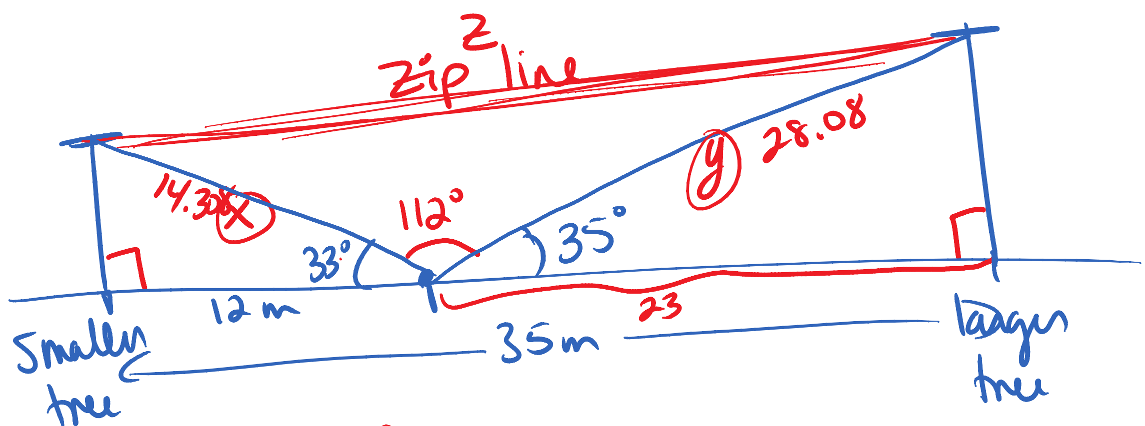


Cosine Law

$$1.84^2 = 0.99^2 + 0.99^2 - 2(0.99)(0.99)\cos\theta$$

$$\theta = 137^\circ$$

13



$$\cos 33^\circ = \frac{12}{x}$$

$$x = \frac{12}{\cos 33}$$

$$x = 14.308$$

$$\cos 35 = \frac{23}{y}$$

$$y = 28.08$$

$$z^2 = 14.308^2 + 28.08^2 - 2(14.308)(28.08)\cos 112^\circ$$

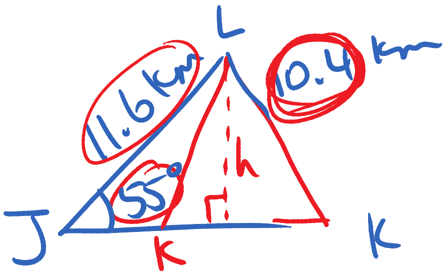
$$z = 36\text{m}$$

More Practice

pg 198 # 1abc, 2ace, 3, 5 ← challenge question

pg 200 # 2bd, 3b, 5, 6a, 8

2c



$10.4 < 11.6$

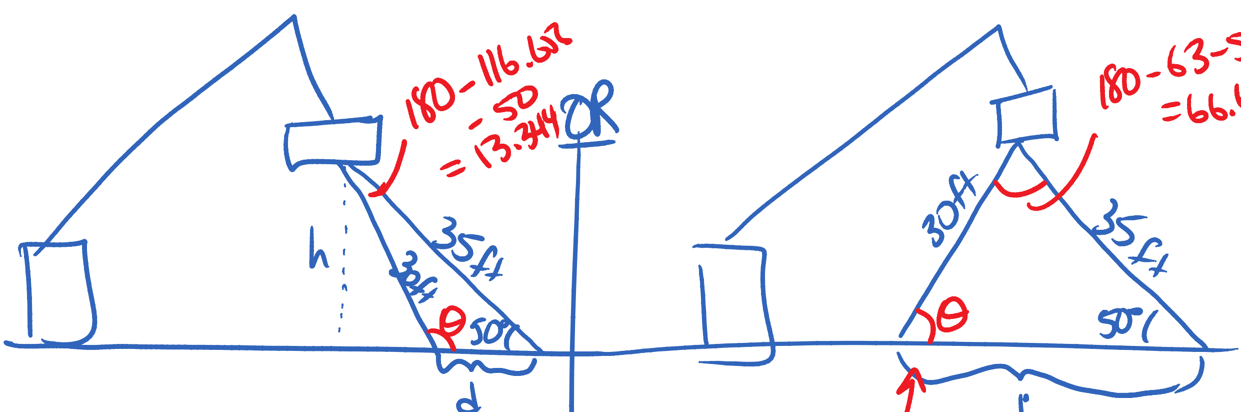
$\sin 55 = \frac{h}{11.6}$

$h = 11.6 \sin 55°$
$= 9.5\text{km}$

$10.4 > 9.5$

$\therefore$ 2 triangles

3



$h = 35 \sin 50° = 26.81$

$30\text{ft} > 26.81$

$\therefore$ 2 triangles

$\frac{\sin\theta}{35} = \frac{\sin 50°}{30}$

$\theta = 63.344$ acute

obtuse

$\hookrightarrow 180 - 63.344$
$= 116.656$

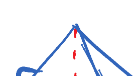
$\frac{d}{\sin 13.344} = \frac{30}{\sin 50}$

$d = 9.0\text{ft}$

$\frac{d}{\sin 66.656} = \frac{30}{\sin 50}$

$d = 36\text{ ft}$

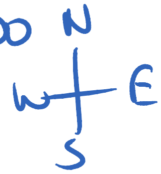
pg200 N

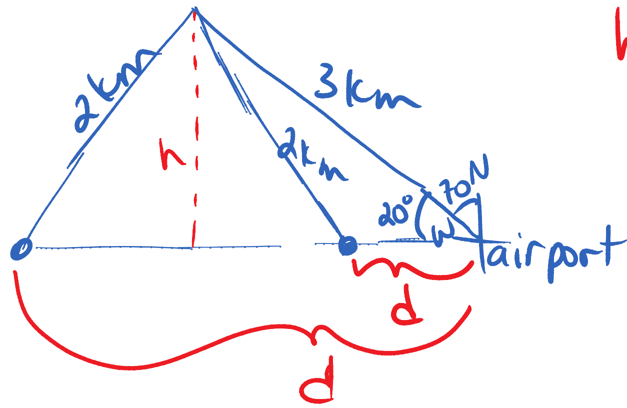


$h = 3\text{km} \sin 20°$

pg200

8





$h = 3\text{km} \sin 20°$

$= 1.026$

$2\text{km} > 1.026$ ← height

$\therefore$ 2 triangles