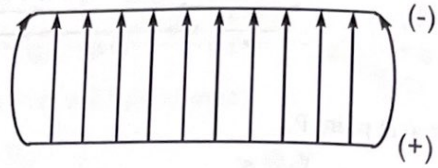
Quiz on Lessons 1 + 2

Lesson ~~4~~ 9 ELECTRIC POTENTIAL IN A UNIFORM ELECTRIC FIELD

NOTES

The electric field between parallel charged plates is uniform.

Look at the electric field between parallel charged plates:



Note that the density of the lines of force is uniform; therefore, the electric field between parallel charged plates is uniform. If this field is uniform, the formula

$$|\vec{E}| = \frac{kq_1}{r^2}$$

cannot describe this field. We need a new formula to describe this uniform electric field.

However, before we describe this field, we must introduce the concept of potential difference. The concept of potential difference is developed from the concepts developed in mechanics (the study of motion and forces that change it).

An object will change its velocity when an unbalanced force acts on it—Newton's First Law of Motion. When a mass is allowed to fall in a gravitational field, the mass will accelerate from a position of high gravitational potential energy to a position of lower gravitational potential energy because of the force of gravity acting on it.

If we want to move a mass from a position of low gravitational potential energy to a position of higher gravitational potential energy, we do work on the mass against gravity. Work done against gravity can be defined in mathematical terms as:

gravity analogy

$$W = F_g d \qquad \text{or} \qquad W = mgh$$

where W = work
F_g = magnitude of the force due to gravity
d = magnitude of the displacement
m = mass
g = magnitude of the gravitational field strength
h = height

The change in gravitational potential energy can also be defined as:

$$E_{p(g)} = mgh$$

NOTES

From this we see that the gravitational potential energy depends on:
- mass of the object ($E_{p(g)} \; \alpha \, m$)
- gravitational field strength ($E_{p(g)} \; \alpha \, g$)
- height the object is moved ($E_{p(g)} \; \alpha \, h$)

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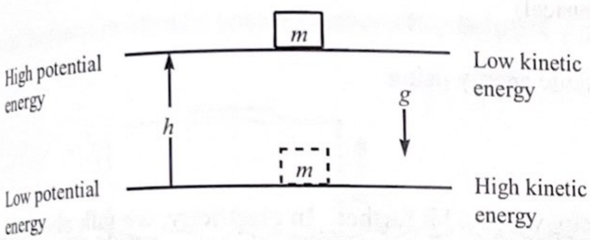
From the Law of Conservation of Energy, the loss in gravitational potential energy of an object becomes kinetic energy.

$E_k = \frac{1}{2}mv^2$



In the same way, when a charged object is allowed to move in a uniform electric field, the charge will accelerate from a position of high electrical potential energy to a position of lower electrical potential energy because of the electric force acting on it. If we want to move a charged object (e.g., a positive particle) in that field (e.g., toward a positive plate), we do work (work against an electric field) on the object. This work can be determined mathematically using the definition of work:

electric

$W = F_e d$

Since $F_e = q|\vec{E}|$

it follows $W = q|\vec{E}|d$

$W = F_e d$
$F_e = q\vec{E}$
$E_p = q\vec{E}d$
and $E_p = q_t V$ $\therefore V = \vec{E}d$

The change in electrical potential energy can also be defined as:

$E_{p(e)} = q|\vec{E}|d$

From this, we see that the electrical potential energy in a uniform field depends on:
- charge of the object ($E_{p(e)} \; \alpha \, q$)
- electric field strength ($E_{p(e)} \; \alpha \, \vec{E}$)
- distance moved parallel to the force ($E_{p(e)} \; \alpha \, d$)

Electric potential energy depends upon:
- charge of object
- electric field
- distance

Again, from the Law of Conservation of Energy, the loss in electrical potential energy of a charged object becomes kinetic energy:

$E_k = \frac{1}{2}mv^2$

NOTES

If $V = \dfrac{\Delta E_{p(e)}}{q}$,

$\Delta E_{p(e)} = qV$,

then the Law of Conservation of Energy, as it applies electric field, becomes

$\Delta E_k = -\Delta E_{p(e)}$ or

$\frac{1}{2}m(v^2 - v_0^2) = -qV$

Note: A negative sign indicates that if kinetic energy is gained, electric potential energy is lost.

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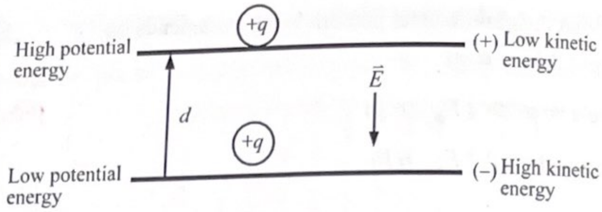
this negative solves the $v = \sqrt{-\ldots}$ problem! (in prev section)

V = electric potential
= potential difference
= voltage!



In electricity, as in mechanics, we calculate work using:

$W = Fd$
or $W = mgh$ (gravitational)
or $W = q|\vec{E}|d$ (electrical)

and we calculate kinetic energy using

$$E_k = \frac{1}{2}mv^2$$

However, in electricity we go a bit further. In electricity, we talk about the electric potential energy per unit charge. (In mechanics, we do not talk about gravitational potential energy per unit mass.)

Energy per unit charge is how we define electric potential.

ELECTRIC POTENTIAL (CONTINUED)

Symbol: V

Definition: electric potential energy per unit charge

When an electric charge is moved in an electric field, the electric potential may change. This change in the electric potential is what we call the potential difference (also called voltage).

Potential difference (also symbolized by V) is the more useful quantity when we are discussing a uniform electric field.

POTENTIAL DIFFERENCE (VOLTAGE)

Symbol: V

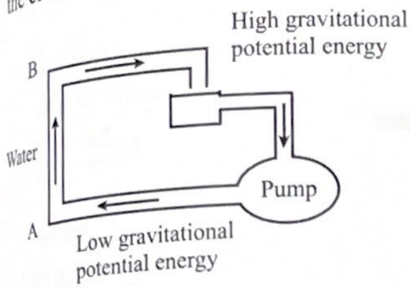
Definition: the change in the electric potential, or the change in the electric potential energy per unit charge.

$$V = \frac{\Delta E_{p(e)}}{q}$$

Note: Electric potential and potential difference are scalar quantities.

NOTES

A battery is a source of potential difference. Just as a water pump will increase the gravitational potential energy of water, a battery will increase the electrical potential energy of a charge.



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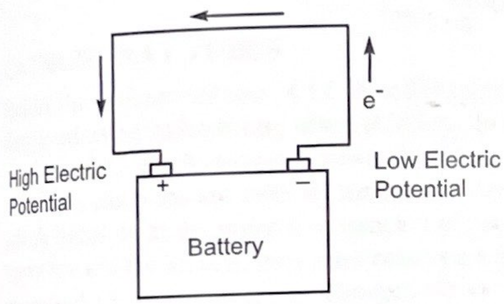
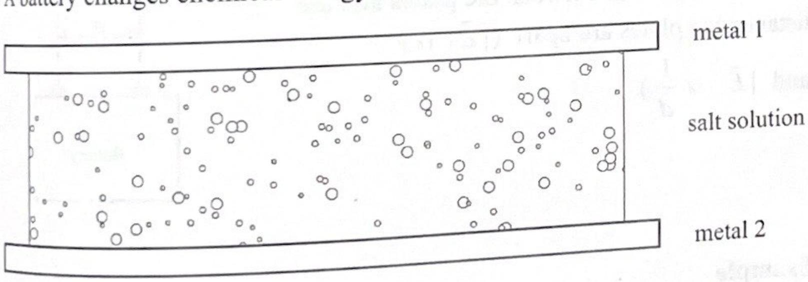
(an increase in electric potential takes place between A and B.)



(an increase in electric potential takes place within the battery.)

The first battery was produced by Alessandro Volta in 1800. The first battery (cell) was a salt solution sandwiched between two different metals. A battery changes chemical energy into electrical energy.



The units for electric field can be expressed as $\frac{N}{C}$ or $\frac{V}{m}$.

Prove to yourself that they are the same.

$W = Fd$

$$\frac{V}{m} = \frac{\frac{J}{C}}{m} = \frac{J}{Cm} = \frac{N\cancel{m}}{C\cancel{m}} = \frac{N}{C}$$

NOTES

We are now able to describe the uniform electric field between parallel plates. This field is described by the formula:

$$|\vec{E}| = \frac{V}{d}$$ — between plates

where

$\vec{E}$ = electric field

V = potential difference between plates

d = distance between plates

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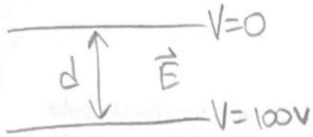


$V = \vec{E}d_{btwn}$

Derivation:

In a uniform electric field,

$$\Delta E_{P(e)} = F_e d \quad (\text{remember } W = \Delta E_{p(e)})$$

or $$\Delta E_{P(e)} = q|\vec{E}|d$$ — distance moved

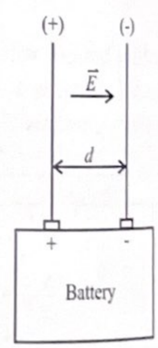
$$\therefore \frac{\Delta E_{P(e)}}{q} = \vec{E}d$$

however, $$V = \frac{\Delta E_{P(e)}}{q}$$

$$\therefore V = |\vec{E}|d$$

or $$|\vec{E}| = \frac{V}{d_{btwn}}$$

Think of the parallel charged plates as extensions of the terminals of a battery. The electric field between parallel plates can be described in terms of the potential difference between the plates and the distance the plates are apart ($|\vec{E}| \; \alpha \; V$ and $|\vec{E}| \; \alpha \; \frac{1}{d}$).



Example

Calculate the electric field strength between two parallel plates that are 6.00×10^{-2} m apart. The potential difference between the plates is 12.0 V.

Solution

$$|\vec{E}| = \frac{V}{d}$$

$$= \frac{12.0 \text{ V}}{6.00 \times 10^{-2} \text{ m}}$$

$$= 2.00 \times 10^{2} \text{ V/M}$$

NOTES

Example

An electron is accelerated from rest through a potential difference of 3.00×10^{4} V. What is the kinetic energy gained by the electron?

Solution

$\Delta E_{k} = -\Delta E_{p(e)}$

$V = \dfrac{\Delta E_{p(e)}}{q}$

$\Rightarrow \Delta E_{p(e)} = qV$

$\therefore \Delta E_{k} = qV$

$= (1.60\times10^{-19}\ \text{C})(3.00\times10^{4}\ \text{V})$

$= 4.80\times10^{-15}\ \text{J}$

$E_k = E_p = qV$

$E_k = qV$

CATHODE-RAY TUBES

Before the widespread use of LCD and plasma screens, video was displayed using cathode-ray tubes (CRTs). In a CRT, electrons are accelerated by an electron gun toward a flat screen enclosed in a vacuum tube. The electrons are initially aimed directly at the centre of the screen, which lights up at the point it is struck with an electron. As electrons travel toward the screen, they pass between a set of two horizontal charged plates and a set of two vertical charged plates. By varying the charge in the plates, electrons can be aimed to specific points across the screen. In this way, electrons are precisely controlled to display images on televisions, computer monitors, and oscilloscopes.

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activity
on beginning
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any energy

PRACTICE EXERCISES

Formulas: $V = \dfrac{\Delta E_{p(e)}}{q_t}$ $\quad |\vec{E}| = \dfrac{V}{d_{\text{between plates}}}$ $\quad E_p = q_t \vec{E} d_{\text{moved}}$ F

1. Two parallel plates are connected to a 12.0 V battery. If the plates are 9.00×10^{-2} m apart, what is the electric field strength between them?

$$\vec{E} = \frac{\Delta V}{d} = \frac{12\text{V}}{9\times10^{-2}\text{m}} = 133\,\frac{\text{V}}{\text{m}}$$ ✓

2. The electric field between two parallel plates is 5.0×10^{3} V/m. If the potential difference between the plates is 2.0×10^{2} V, how far apart are the plates?

$$d = \frac{\Delta V}{\vec{E}} = \frac{2\times10^{2}\,\text{V}}{5\times10^{3}\,\text{V/m}} = 4.0\times10^{-2}\,\text{m}$$ ✓

3. Two parallel plates are 7.3 cm apart. If the electric field strength between the plates is 2.0×10^{3} V/m, what is the potential difference between the plates?

0.073 m

$$\Delta V = \vec{E}d = (2\times10^{3}\,\tfrac{\text{V}}{\text{m}})(0.073\,\text{m})$$
$$= 1.5\times10^{2}\,\text{V}$$ ✓

4. An alpha particle gains 1.50×10^{-15} J of kinetic energy. Through what potential difference was it accelerated?

$$V = \frac{E_k}{q} = \frac{1.5\times10^{-15}\,\text{J}}{3.2\times10^{-19}\,\text{C}} = 4.69\times10^{3}\,\text{V}$$ ✓

5 here now

5. A proton is accelerated by a potential difference of 7.20×10^2 V. What is the change in kinetic energy of the proton?

$$\Delta E_k = Vq$$
$$= (7.2\times10^2 V)(1.6\times10^{-19} C)$$
$$= \underline{1.15\times10^{-16}\ J}\ \checkmark$$

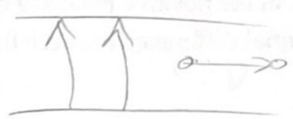
6. What maximum speed will an alpha particle reach if it moves from rest through a potential difference of 7.50×10^3 V?

$$E_k = Vq$$
$$\frac{1}{2}mv^2 = Vq$$
$$v = \sqrt{\frac{2Vq}{m}} = \sqrt{\frac{2(7.5\times10^3)(3.2\times10^{-19}C)}{(6.65\times10^{-27}kg)}} = \underline{8.50\times10^5\ m/s}\ \checkmark$$

7. A proton is placed in an electric field between two parallel plates. If the plates are 6.0 cm apart and have a potential difference between them of 7.50×10^1 V, how much work is done against the electric field when the proton is moved 3.0 cm parallel to the plates?

$$W = \Delta E_p = \frac{V}{d}$$

0 work since parallel motion, (no change in E_p) ✓



8. In the previous question, how much work would be done against the electric field if the proton was moved 3.0 cm perpendicular to the plates?

$$W = q\overset{F}{\overbrace{E}} d_{moved} = \frac{(1.6\times10^{-19})(7.5\times10^1)(0.03)}{(0.06m)} = \underline{6.0\times10^{-18}\ J}\ \checkmark$$

$$E = \frac{V}{d_{btwn}}$$

⑨ here now

$F \propto \frac{1}{r^2} \rightarrow \frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$

9. A charged particle was accelerated from rest by a potential difference of 4.20×10^2 V (V_1). If this particle increased its kinetic energy to 3.00×10^{-17} J (E_{k1}), what potential difference would be needed to increase the kinetic energy of the same particle to 9.00×10^{-17} J? (E_{k2})

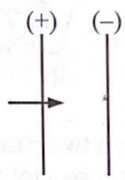
$\Delta V = \frac{\Delta E_p}{Q}$

$E_k \propto V$

$\frac{E_{k1}}{E_{k2}} = \frac{V_1}{V_2}$

$V_2 = \frac{V_1 E_{k2}}{E_{k1}} = \frac{(4.2 \times 10^2\ V)(9 \times 10^{-17}\ J)}{(3 \times 10^{-17}\ J)} = 1.26 \times 10^3\ V$ ✓

10. An alpha particle with an initial speed of 7.15×10^4 m/s enters through a hole in the positive plate between two parallel plates that are 9.00×10^{-2} m apart, as shown above. If the electric field between the plates is 1.70×10^2 V/m, what is the speed of the alpha particle when it reaches the negative plate?



@ "+" plate = @ "–" plate

$E_p + E_k = E_k$

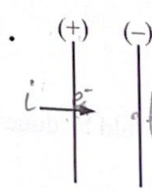
$q\vec{E}d + \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2$

$(3.2 \times 10^{-19}\ C)(1.7 \times 10^2\ \frac{V}{m})(9 \times 10^{-2}\ m) + \frac{1}{2}(6.65 \times 10^{-27}\ kg)(7.15 \times 10^4\ m/s)^2 = \frac{1}{2}(6.65 \times 10^{-27})v_f^2$

$v_f = 8.11 \times 10^4$ m/s ✓

11. An electron with a speed of 5.0×10^5 m/s enters through a hole in the positive plate and collides with the negative plate at a speed of 1.0×10^5 m/s. What is the potential difference between the plates?

$V = ?$



$E_p + E_k = E_k$

$Vq + \frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2$

$V = \frac{\frac{1}{2}(9.11 \times 10^{-31}\ kg)(5 \times 10^5\ m/s)^2 - \frac{1}{2}(9.11 \times 10^{-31})(1 \times 10^5\ m/s)^2}{1.6 \times 10^{-19}}$

$= +0.68$ V ✓

(12) here now

12. The electric field strength between two parallel plates is 9.3×10^{2} V/m when the plates are 7.0 cm apart. What would the electric field strength be if the plates were 5.0 cm apart?

E_1, d_1, d_2

$E=\frac{V}{d}$ $\vec{E} \propto \frac{1}{d}$ $\frac{E_2}{E_1}=\frac{d_1}{d_2}$

$E_2=\frac{(0.07\,m)(9.3\times10^{2}\,V/m)}{(0.05\,m)} = 1.3\times10^{3}\,\frac{V}{m}$ ✓

13. What is the electric field strength 1.00 cm from the positive charged plate if the parallel plates are 5.00 cm apart and the potential difference between the plates is 3.00×10^{2} V?

$\vec{E}=\frac{V}{d}=\frac{3\times10^{2}\,V}{0.05\,m}=6.00\times10^{3}\,\frac{V}{m}$ ✓

$\vec{E}$ is same everywhere between plates

14. Through what potential difference must an electron be accelerated from rest to give it a speed of 6.00×10^{6} m/s?

$E_p = Vq = E_k$

$V=\frac{\frac{1}{2}mv^2}{q}=\frac{\frac{1}{2}(9.11\times10^{-31}\,kg)(6\times10^{6}\,m/s)^2}{1.6\times10^{-19}\,C}$

$= 102\,V$ ✓

15. If an alpha particle is accelerated from rest through a distance of 4.00 cm by a uniform electric field in 2.50×10^{-5} s, what is the electric field strength?

$v_{avg}=\frac{0.04\,m}{2.5\times10^{-5}\,s}$

$=1600\,m/s \rightarrow v_f = 1600 \times 2 = 3200$

why?

$E_k=\frac{1}{2}mv^2$

$=\frac{1}{2}(6.65\times10^{-27}\,kg)(3200\,m/s)^2$

$=3.4048\times10^{-20}\,J$

$\vec{E}=\frac{(V)}{d}=\frac{(E_p)}{q}\frac{1}{d}$

$=\frac{(E_k)}{qd}$

$=\frac{3.4048\times10^{-20}}{(3.2\times10^{-19}\,C)(0.04\,m)}$

$=2.66\,\frac{N}{C}$

$E=\frac{F_e}{q_\alpha}=\frac{ma}{q_\alpha}$

$a=$
$v_i=0$
$t=2.5\times10^{-5}$
$d=0.04\,m$

$d=\cancel{v_i t}+\frac{1}{2}at^2$

$\vec{a}=\frac{2d}{t^2}=1.28\times10^{8}\,m/s^2$

$\vec{F}_e=m\vec{a}=(6.65\times10^{-27}\,kg)(1.28\times10^{8}\,m/s^2)$
$=8.512\times10^{-19}\,N$

$\vec{E}=\frac{\vec{F}_e}{q_\alpha}=\frac{8.512\times10^{-19}\,N}{3.2\times10^{-19}\,C}$

$=2.66\,\frac{N}{C}$

☆ ☆

16 + 17 here

16. What is the potential difference between two parallel charged plates that are 7.50 cm apart if a force of 5.30×10^{-14} N is needed to move an alpha particle from the negative plate to the positive plate?

$\vec{F} = \vec{E}q$

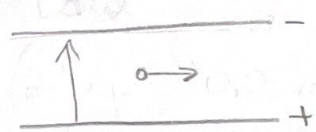
$F = \frac{\Delta V}{d} q$

or (work) $V = \frac{E_p}{q} \Rightarrow \Delta V = \frac{Fd}{q} = \frac{(5.3\times10^{-14}\,N)(0.075\,m)}{3.2\times10^{-19}\,C} = 1.24\times10^{4}\,V$ ✓

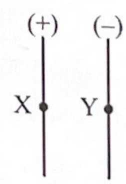
17. An electric field of 2.40×10^{2} N/C is produced by two horizontal parallel plates set 4.00 cm apart. If a charged particle of 2.00 μC is moved 3.00 cm perpendicular to the electric field, what is the work done against the electric field?

↳ so distance from plate doesn't change so no work is done ✓

$W = \Delta E_p$



18. A proton accelerates from rest from plate X to plate Y at the same time as an electron accelerates from rest from plate Y to plate X. If the potential difference between the two plates is 60.0 V,



a) what is the speed of the proton when it reaches plate Y?

$E_k = E_p$

$\frac{1}{2}mv^2 = Vq$

$v = \sqrt{\frac{2Vq}{m}} = \sqrt{\frac{2(60\,V)(1.6\times10^{-19})}{1.67\times10^{-27}\,kg}} = 1.07\times10^{5}\,m/s$ ✓

b) what is the speed of the electron when it reaches plate X?

$v = \sqrt{\frac{2(60\,V)(1.6\times10^{-19})}{9.11\times10^{-31}\,kg}} = 4.59\times10^{6}\,m/s$ ✓

So much faster because lower mass.

that are 7.50 cm apart if a force
ive plate to the positive plate?

1.24×10^4 V ✓

allel plates set 4.00 cm apart.
e electric field, what is the work

istance from
late doesn't
change so
no work is done ✓

e as an electron accelerates
ne two plates is 60.0 V,

$.07 \times 10^5$ m/s ✓

M/s ✓

use lower mass.

19. A charged particle was accelerated from rest by a potential difference of 2.50×10^5 V. If the particle reached a maximum speed of 2.90×10^4 m/s, what potential difference would be required to accelerate this particle from rest to a velocity of 7.25×10^4 m/s?

$E_k = E_p$ $\quad v^2 \propto V$ $\quad \frac{v_1^2}{v_2^2} = \frac{V_1}{V_2}$ $\quad V_2 = \frac{V_1 v_2^2}{v_1^2}$

$\frac{1}{2}mv^2 = VQ_t$

$= \frac{(2.5\times10^5)(7.25\times10^4)^2}{(2.9\times10^4)^2}$

$= 1.56\times10^6$ V ✓

20. An electron is accelerated from rest through a potential difference of 5.00×10^3 V. What is the resulting speed of this electron?

$\frac{1}{2}mv^2 = VQ_t$

$v = \sqrt{\frac{2VQ_t}{m}} = \sqrt{\frac{2(5000)(1.6\times10^{-19})}{9.11\times10^{-31}}} = 4.19\times10^7$ m/s ✓

21. An alpha particle is placed between two horizontal parallel charged plates that are 2.00 cm apart. The potential difference between the plates is 12.0 V.

a) What is the electric force acting on the alpha particle?

$F_e = \vec{E}Q_t$
$= \frac{V}{d}Q_t = \frac{12}{0.02}(3.2\times10^{-19}) = 1.92\times10^{-16}$ N ✓

b) What is the gravitational force acting on the alpha particle?

$F_g = mg = (6.65\times10^{-27}\text{ kg})(9.81\frac{N}{kg}) = 6.52\times10^{-26}$ N ✓

c) If it is assumed that the electric force and the gravitational force are acting in opposite directions, what is the net force acting on the alpha particle?

1.92×10^{-16} N $- 6.52\times10^{-26}$ N $= 1.92\times10^{-16}$ N ✓

d) What is the acceleration of the alpha particle?

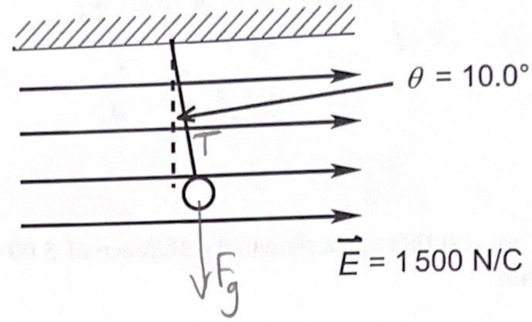
$F = ma \quad a = \frac{F}{m} = \frac{1.92\times10^{-16}\text{ N}}{6.65\times10^{-27}} = 2.89\times10^{10}$ m/s² ✓

e) What potential difference would be required between the plates so that the alpha particle becomes suspended?

need $F_g = F_e = \frac{V}{d}Q_t \quad V = \frac{dF_g}{Q_t} = \frac{(0.02)(6.52\times10^{-26}\text{ N})}{(3.2\times10^{-19})}$

$= 4.08\times10^{-9}$ V ✓

22. A 0.50 kg ball is suspended in a uniform electric field ($|\vec{E}|$ = 1 500 N/C) as shown in the diagram below. ✓



$\tan 10° = \frac{F_e}{F_g}$

Calculate the charge on the ball.

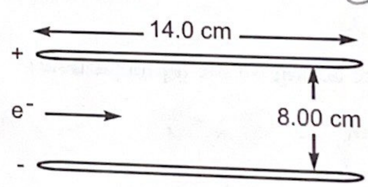
$F_e = F_g \tan 10°$
$= 0.5(9.81)\tan 10°$
$= 0.864884$ N

$\vec{E} = \frac{\vec{F_e}}{Q_t}$

$Q_t = \frac{F_e}{E} = \frac{0.86488\text{ N}}{1500\text{ N/C}} = \underline{5.8 \times 10^{-4}}$ C

23. An electron travelling horizontally at a speed of 8.70×10^6 m/s enters an electric field of 1.32×10^3 N/C between two horizontal parallel plates as illustrated below.

F_g is insignificant
$F_g = m_e \times 9.8$
$\approx 9.11 \times 10^{-30}$ N
compared to F_e @ $\times 10^{-16}$ N



① $V = 8.7 \times 10^6$ m/s
$t = ?$
$d = 0.14$ m
$t = \frac{d}{V} = \frac{0.14}{8.7 \times 10^6}$
$= 1.6092 \times 10^{-8}$ s

Calculate the magnitude of the vertical displacement of the electron as it travels between the plates.

② $F_e = EQ$
$= (1.32 \times 10^3 \text{ N/C})(1.6 \times 10^{-19})$
$= 2.112 \times 10^{-16}$ N
$= ma$

③ $a = \frac{F}{m} = \frac{2.112 \times 10^{-16}\text{ N}}{9.11 \times 10^{-31}\text{ kg}}$
$= 2.31833 \times 10^{14}$ m/s²

④ $V_{vi} = 0$
$a = 2.31833 \times 10^{14}$ m/s²
$d = ?$
$t = 1.6092 \times 10^{-8}$ s

$d = \cancel{v_0 t} + \frac{1}{2}at^2$
$= \frac{1}{2}(2.31833 \times 10^{14})(1.6092 \times 10^{-8})^2$
$= 3.00 \times 10^{-2}$ m ✓
$= 3.00$ cm