6.4 Optimization: Creating a Model

Optimization: Creating a Model [6.4]

Example 1:

Three teams are travelling to a basketball tournament in cars and minivans.

clues

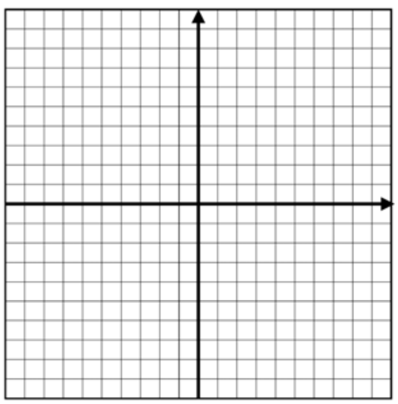
- Each team has no more than 2 coaches and 14 athletes. = 16 people × 3 teams = 48 (no more)
- Each car can take 4 team members, and each minivan can take 6 team members.
- No more than 4 minivans and 12 cars are available. → Constraints

The school wants to know the combinations of cars and minivans that will require the minimum and maximum number of vehicles. Create a model to represent this situation.

Objective function

$V = x + y$

# of vehicles



# of cars = x

# of minivans = y

restrictions

$x, y \in W$

Inequalities

$x \leq 12$

$y \leq 4$

$4x + 6y \leq 48$

graph these

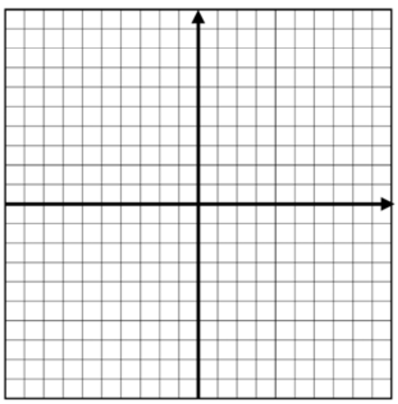
Example 2:

A refinery produces oil (x) and gas (y).

- At least 2L of gas (y) is produced for each litre of oil (x). $\boxed{y \geq 2x}$
- The refinery can produce up to 9 million litres of oil and 6 million litres of gas each day.
- Gasoline is projected to sell of $1.10 per litre. Oil is projected to sell for $1.75 per litre.

The company needs to determine the daily combination of gas and oil that must be produced to maximize revenue. Create a model to represent the situation.

x = # of l of oil
y = # of l of gas



$\boxed{x \leq 9\,000\,000}$

$\boxed{y \leq 6\,000\,000}$

graph

Optimization / Objective function

$R = \$1.75x + \$1.10y$ ← don't graph - just use to calc. max and min

revenue $

Example 3: YOUR TURN!

$x = \#$ $y = \#$ $x, y \in W$

A toy company manufactures two types of toy vehicles: racing cars and sport-utility vehicles.

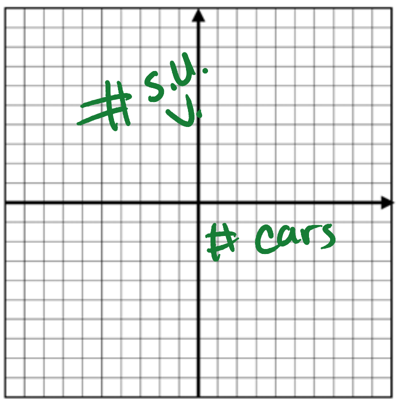
- Because the supply of materials is limited, no more than 40 racing cars and 60 sport-utility vehicles can be made each day. $\boxed{x \le 40}$ $\boxed{y \le 60}$
- However, the company can make 70 or more vehicles, in total, each day. $\boxed{x + y \ge 70}$ graph
- It costs \$8 to make a racing car and \$12 to make a sport-utility vehicle.

The company wants to know what combinations will result in the minimum and maximum costs. C
Create a model to represent the situation.

Objective function

$C = \$8x + \$12y$

\$



6.4 Homework pg 330

#3 x = # of pops $x, y \in W$

y = # of juice Stippled

graph $\boxed{\begin{aligned} x+y &\le 240 \\ y &\ge 2x \end{aligned}}$

pop

2 cans of juice for each pop

Objective function

$$R = \$1.25x + \$1y$$

7 x = # of hectares of barley

y = # " " " wheat

$\boxed{\begin{aligned} x+y &\le 1000 \\ y &\ge 3x \end{aligned}}$ ← graph

wants more wheat than barley

Objective Function

$$R = \$\underset{\text{wheat}}{5.25(50y)} + \$\underset{\text{barley}}{3.61(38x)}$$

\$

6.5 Optimization: Exploring Solutions

Optimization: Exploring Solutions [6.5]

Example 1:

Consider the situation: Restrictions: $x \in R,\ y \in R$ shading, solid lines

Constraints: $x + 3y \leq 9, x - y \leq 3, x \geq -3$ ← graph

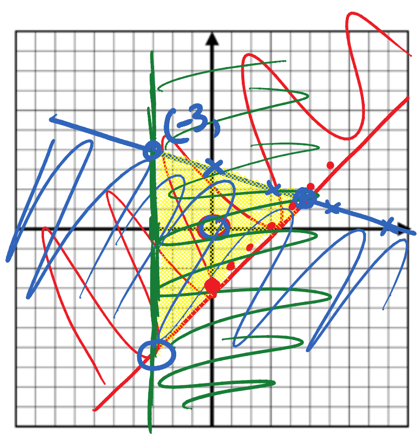
Objective function: $P = 2x + y$ ← don't graph, use to find max + min

a.) Draw a graph to model the situation.

b.) What point in the feasible region would result in the maximum value of the objective function?

c.) What point in the feasible region would result in the minimum value of the objective function?

a)



$x+3y \leq 9$ test pt (0,0)

$3y \leq -x+9$ $\quad 0 \overset{?}{\leq} 9$ yes

$y \leq -\frac{1}{3}x + 3$

$x - y \leq 3$ test pt (0,0)

$-y \leq -x+3$ $\quad 0 \overset{?}{\leq} 3$ yes

$y \geq x - 3$

$x \geq -3$

Solution area

b/c) $P = 2x + y$ ← objective function

-plug in coordinates of vertices

(-3,4) (4.5, 1.5) (-3, -6)

$P = 2(-3) + 4 = -2$

$P = 2(4.5) + 1.5 = 10.5$ maximum

$P = 2(-3) + -6 = -12$ minimum

Example 2:

A toy company manufactures two types of toy vehicles: racing cars and sport-utility vehicles.

- Because the supply of materials is limited, no more than 40 racing cars and 60 sport-utility vehicles can be made each day.
- However, the company can make 70 or more vehicles, in total, each day.
- It costs $8 to make a racing car and $12 to make a sport-utility vehicle.

The company wants to know what combinations will result in the minimum and maximum costs, and what those costs will be.

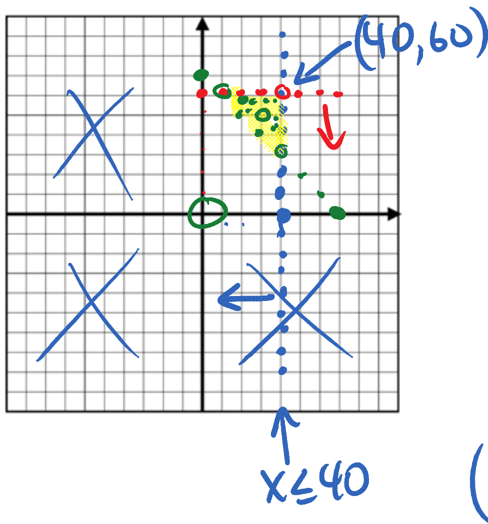
x y

$x \le 40$ ✓ $y \le 60$ ✓ graph

$x + y \ge 70$

$C = 8x + 12y$ Obj. func.

$x, y \in W$



$0 + 0 \overset{?}{\ge} 70$ No

vertices of soln area

(40, 60) (10, 60) (40, 30)

$C = 8(40) + 12(60)$ $8(10) + 12(60)$ $C = 8(40) + 12(30)$

\$1040 \$800 = \$680

max min

Practice pg 334 # 1-3

③ a) Check vertices, look for largest # of books

(50, 200)

b) No, twice as many novels as cookbooks

c) (50 cookbooks, 100 novels)

d) (150, 200) $\quad W = 0.5(50) + 0.25(200)$

$= 75''$

e) (0,0) $\quad W = 0.5(0) + 0.25(0)$

$= 0''$

Pg 341 # 1, 5, 9, 11, 13

9 d) # of bags of walnuts $= y$

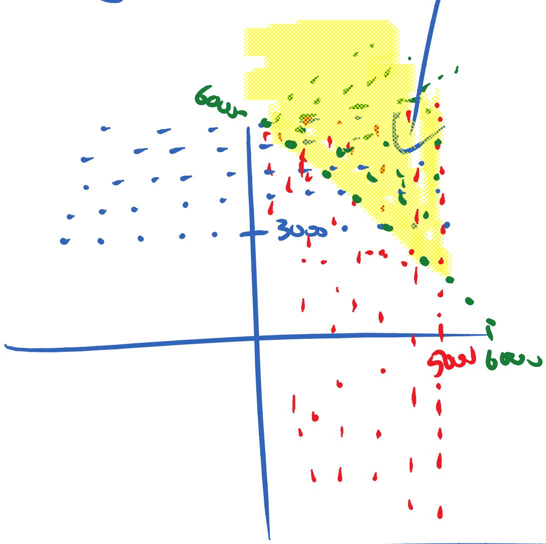
# of bags of almonds $= x$

$$\left\{\begin{array}{l} y \geq 3000 \\ x \leq 5000 \\ x + y \geq 6000 \end{array}\right\} a$$

$x, y \in W$

e) Objective Function:

$C = \$11.19(x) + \$13.10(y)$



13

$x \geq 12$ @ \$8.75

$y \leq 24$ @ \$9

$x + y \leq 32$

$x, y \in W$

Objective Function

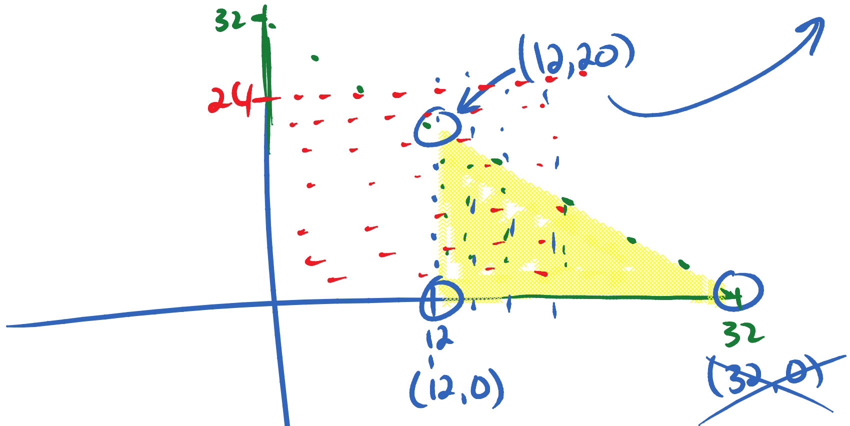
graph

$x, y \in W$

stipple

Objective function

$E = \$8.75x + \$9y$



$(12, 20)$

$8.75(12) + 9(20)$

$=$

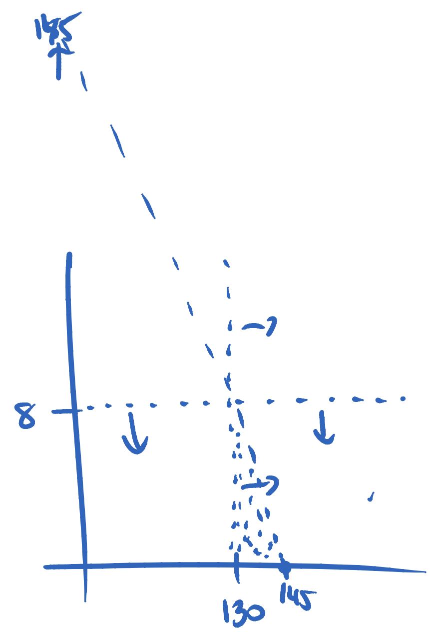
~~(12,0)~~

(11) $x, y \in W$

economy

busines

$$\left.\begin{aligned} x + y &\le 145 \\ x &\ge 130 \\ y &\le 8 \end{aligned}\right\}$$



6.6 Optimization: Programming - Key

Optimization: Linear Programming [6.6]

Example 1:

The following model represents an optimization problem. Determine the maximum solution.

Optimization Model

colour everything all Quadrants solid lines

Restrictions: $x \in R$ and $y \in R$

Constraints: $y \leq 1, 2y \geq -3x + 2, y \geq 3x - 8$

Objective function: $D = -4x + 3y$

① $y \leq 1$ → horizontal line $y = 1$

② $2y \geq -3x + 2$ → Test (0,0)

$y \geq -\frac{3}{2}x + 1$

$2y \geq -3x + 2$

$2(0) \geq -3(0) + 2$

$0 \geq +2$ ✗

False

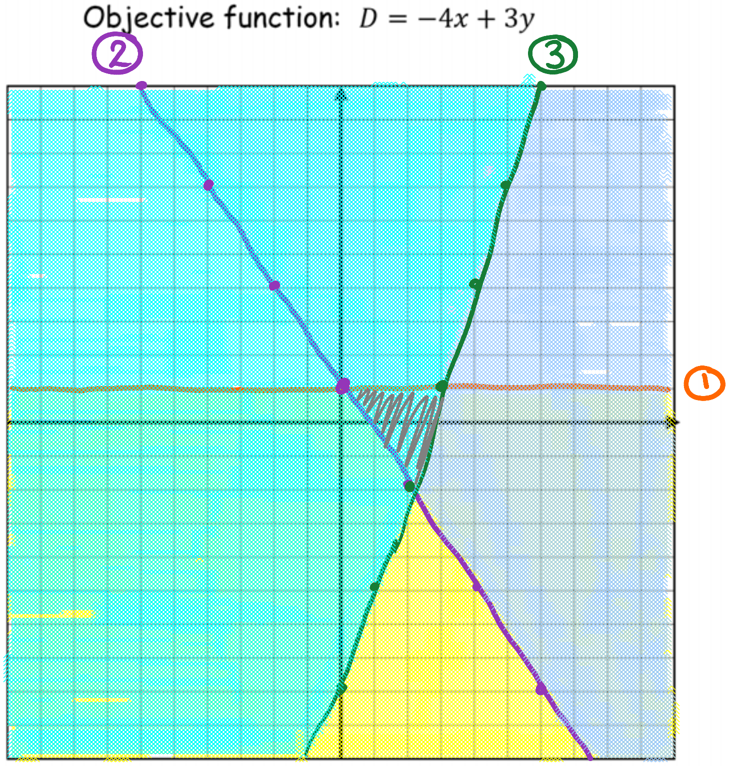
③ $y \geq 3x - 8$ → Test (0,0)

$y \geq 3x - 8$

$0 \geq 3(0) - 8$

$0 \geq -8$ ✓

True



→ What are the 3 verticies of our feasible region?

(0,1) (3,1) (2,-2)

→ Now put them in the Objective Function

(0,1)

$D = -4x + 3y$

$D = -4(0) + 3(1)$

$\boxed{D = 3}$

maximum

(3,1)

$D = -4x + 3y$

$D = -4(3) + 3(1)$

$D = -12 + 3$

$D = -9$

(2,-2)

$D = -4x + 3y$

$D = -4(2) + 3(-2)$

$D = -8 - 6$

$D = -14$

Example 2:

real items
Whole numbers
only Quadrant I

Larry and Tony are baking cupcakes and banana mini-loaves to sell at a school fundraiser.

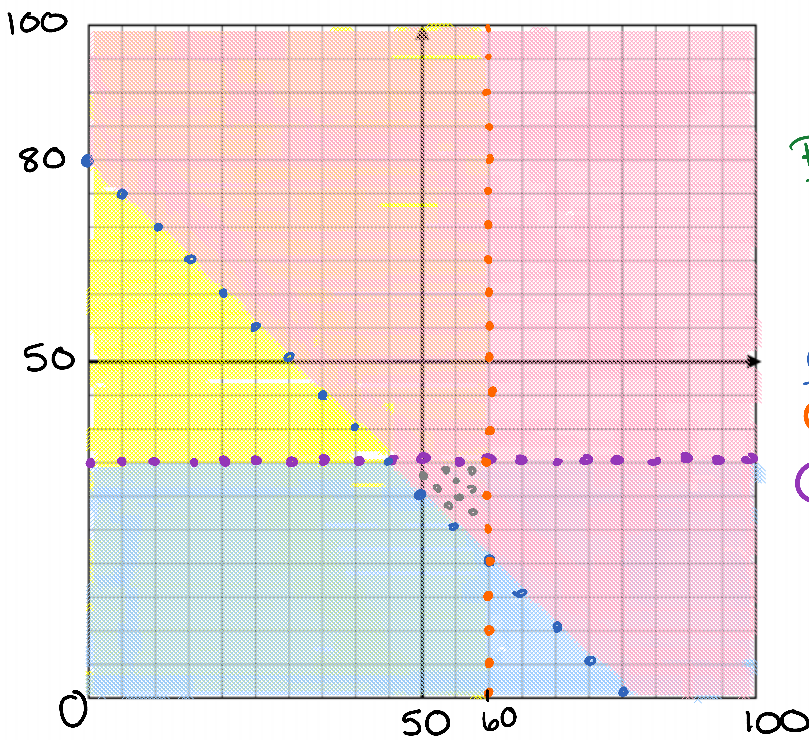
less than / equal to

- No more than 60 cupcakes and 35 mini-loaves can be made each day
- Larry and Tony can make more than 80 baked goods, in total, each day.
- It costs \$0.50 to make a cupcake and \$0.75 to make a mini-loaf. → objective function

Constraints

$C = .50x + 0.75y$

They want to know the minimum costs to produce the baked goods.



Options:
Let cupcakes = x
mini-loaves = y

Restrictions

$x \in W$
$y \in W$

Constraints

① $x \leq 60$ colour left

② $y \leq 35$ colour below

$x + y \geq 80$ → Test (0,0)

③ $y \geq -x + 80$

$x + y \geq 80$
$0 \geq 80$
× False

Input verticies into Objective Function > $C = 0.50x + 0.75y$

(45,35)
$C = 0.50x + 0.75y$
$C = 0.5(45) + 0.75(35)$
$C = \$48.75$

(60,35)
$C = 0.50x + 0.75y$
$C = 0.50(60) + 0.75(35)$
$C = \$56.25$

(60,20)
$C = 0.50x + 0.75y$
$C = 0.50(60) + 0.75(20)$
$C = \$45.00$

minimum costs

Example 3:

L&G Construction is competing for a contract to build a fence. → Real Fence Whole Numbers → dots Only Quadrant I

- The fence will be no longer than 50yd and will consist of narrow boards that are 6in wide and wide boards that are 8in wide. (less than ≤ 50)
- There must be no fewer than 100 wide boards and no more than 80 narrow boards.
- The narrow boards cost \$3.56 each, and the wide boards cost \$4.36 each. → Objective function info

Determine the maximum and minimum costs for the lumber to build the fence.

Options

Let narrow boards = x
wide boards = y

Restrictions: $x \in W$
$y \in W$

Constraints

① $y \geq 100$ colour above

② $x \leq 80$ colour left

$6x + 8y \leq 50$ yards

inches BUT

so... 1 yard = 36"

50 x 36" = 1800"

$6x + 8y \leq 1800$

$8y \leq -6x + 1800$

③ $y \leq -\frac{3}{4}x + 225$

Test (0,0)

$6x + 8y \leq 1800$

$0 \leq 1800$ ✓ True

Not a triangle > Vertices

Input into $C = 3.56x + 4.36y$

(0, 100) → \$436.00 → Minimum Cost (narrow, wide)

(0, 225) → \$981.00

(80, 100) → \$720.80

(80, 165) → \$1004.20 → Max Cost

Narrow, wide