5.1

In Summary

Key Ideas

- Knowing the probability of an event is useful when making decisions.
- The **experimental probability** of event A is represented as

$$P(A) = \frac{n(A)}{n(T)}$$

where $n(A)$ is the number of times event A occurred and $n(T)$ is the total number of trials, T, in the experiment.
- The **theoretical probability** of event A is represented as

$$P(A) = \frac{n(A)}{n(S)}$$

where $n(A)$ is the number of favourable outcomes for event A and $n(S)$ is the total number of outcomes in the sample space, S, where all outcomes are equally likely.
- A game is fair when all the players are equally likely to win.

Need to Know

- An event is a collection of outcomes that satisfy a specific condition. For example, when throwing a regular die, the event "throw an odd number" is a collection of the outcomes 1, 3, and 5.
- The probability of an event can range from 0 (impossible) to 1 (certain). You can express probability as a fraction, a decimal, or a percent.
- You can use theoretical probability to determine the likelihood that an event will happen.

#4

sum/prod	1	2	3	4
1	2/1	3/2	4/3	5/4
2	3/2	4/4	5/6	6/8
3	4/3	5/6	6/9	7/12
4	5/4	6/8	7/12	8/16

sum wins $\frac{7}{16}$

product wins $\frac{8}{16}$

FURTHER Your Understanding

1. How can you change Sasha's game so that it is fair? Explain. reverse rules each turn
   have 1 – 4 and find even/odd sums 8/16, 8/16
2. Consider each game below. Is it fair? If it is not fair, which player has the advantage? Explain.
   a) Matt and Pat each toss a coin. If the coins land as both heads or both tails, Matt wins. If the coins land as a head and a tail, Pat wins.

	H	T
H	HH	HT
T	HT	TT

   Fair

   b) Treena, Lena, and Gina each toss a coin. If all three coins land as heads, Treena wins. If all three coins land as tails, Lena wins. Otherwise, Gina wins. No, Gina will win more often
   c) Ann and Dan each roll a die. If the sum of the two dice is greater than 7, Ann wins. If the sum is less than 7, Dan wins. If the sum is 7, they tie.

Sum	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

   Fair
   15 less
   15 more
3. Everard says he has a 120% chance of making the school football team. Is this possible? Explain. No, can't be more chances than positions.
4. Mika suggests playing Sasha's game with four slips of paper, numbered 1 to 4. Is Sasha's game now fair? If not, who has the advantage? Explain.
   See above   No   product Player 2

5.2

CHECK Your Understanding

1. The odds in favour of Marcia passing her driver's test on the first try are 5 : 3.
   a) Determine the odds against Marcia passing her driver's test. 3 : 5
   b) Determine the probability that she will pass her driver's test. $\frac{5}{8}$ = .625

2. Colby has 10 coins in his pocket, and 3 of these coins are loonies. He reaches into his pocket and pulls out a coin at random.
   a) Determine the probability of the coin being a loonie. $\frac{3}{10}$ = 0.3
   b) Determine the odds against the coin being a loonie. 7 : 3

3. Lily draws a card at random from a standard deck of 52 playing cards.
   a) Determine the probability of the card being red. 26/52 = 0.5
   b) Determine the odds in favour of the card being red. 26 : 26 1 : 1
   c) Determine the odds against the card being a spade. 39 : 13 3 : 1
   d) Determine the probability of the card being a face card. 12/52 = 0.231

PRACTISING

The word "risk" comes from the Latin word for cliff.

4. Mina notices that apple juice is on sale at a local grocery store. The last five times that apple juice was on sale, it was available only twice.
   a) Determine the odds in favour of apple juice being available this time. 2 : 3
   b) Determine the odds against apple juice being available this time. 3 : 2

5. There are 30 students in Mario's Grade 12 math class. The odds in favour of two students sharing a birthday are 7 : 3. Determine the probability of two students sharing a birthday. $\frac{7}{10}$ = 0.7

6. Jamia likes to go wall climbing with her friends. In the past, Jamia has climbed to the top of the wall 12 times in 24 attempts.
   a) Determine the probability of Jamia climbing to the top this time. $\frac{12}{24}$ = 0.
   b) Determine the odds against Jamia climbing to the top. 12 : 12 1 : 1
   c) The odds from part b) are called "even odds." Explain what this term might mean. 50/50 chance → 1 : 1

7. The weather forecaster says that there is a 60% probability of snow tomorrow. What are the odds against snow? 60/100 40 : 60 → 2 : 3

8. About 8% of men and 0.5% of women see no difference between the colours red and green. These people are often useful in the military because they can detect khaki camouflage much better than people who do see a difference between red and green. What are the odds in favour of Allan being able to detect camouflage?

$P = \frac{8}{100}$ 8 : 92 → 2 : 23

odds probability

9. Katherine plays ringette. She has scored 4 times in 20 shots on goal. She says that the odds in favour of her scoring are 1 to 5. Is she right? Explain. no

4:16 = 1:4. $\frac{1}{5} = 0.2$ ✓

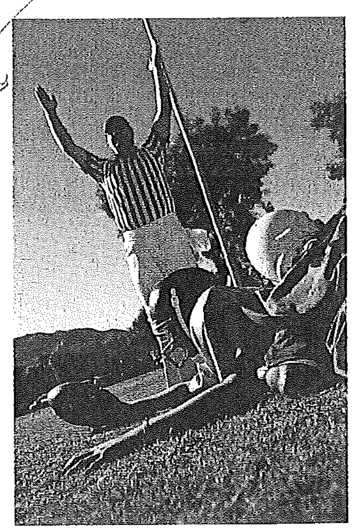
10. Jason has been awarded a penalty shot in a hockey game. Gilles is the goalie. Jason has scored 5 times in his last 10 penalty shots. Gilles has blocked 8 of the last 10 penalty shots.
   a) Determine the odds in favour of Jason scoring, using his data. 5:5 → 1:1 ✓
   b) Determine the odds in favour of Jason scoring, using Gilles' data. 2:8 → 1:4
   c) Explain why your answers to parts a) and b) are different. considering diff people, doing diff things

11. A survey in a Western Canadian city determined that the odds in favour of a person between 18 and 35 using a social networking site are 31:19. Determine the probability of a randomly selected person between 18 and 35 using a social networking site. $\frac{31}{50} = 0.62$ ✓

12. The coach of a basketball team claims that, for the next game, the odds in favour of the team winning are 3:2, the odds in favour of the team losing are 1:4, and the odds against a tie are 4:1. Are these odds possible? Explain.

13. Ratings for the program *Show Trial* indicate that 35% of the viewers are female, 65% are male, 30% are under 18, 20% are 19 to 30 years old, 10% are 30 to 45 years old, and 40% are older than 45. Suppose that someone is watching *Show Trial*.
   a) What are the odds in favour of this person being male?
   b) What are the odds in favour of this person being older than 45?

14. In a study, 70% of the people who were vaccinated did not get sick, and 42% of the people who were not vaccinated did get sick. (so 58 did not get sick)
   a) What are the odds against getting sick if you are vaccinated? 70:30 → 7:3 → c) 49:21
   b) What are the odds against getting sick if you are not vaccinated? 58:42 → 29:21
   c) Express the odds against from parts a) and b) with the same second term. ✓
   d) Should you be vaccinated? Explain. more chance to stay healthy if vaccinated



15. A high-school football team has the ball at the opponent's 2 yd line. It is the third down. The team is behind by 3 points, with only one second left in the game. The players have two options:
   - They can try to score a touchdown. In the past, they have succeeded 5 out of 12 times. If they score a touchdown, they will win the game.
   - They can try to kick a field goal. The kicker has scored a field goal from 20 yd or less in 5 of 6 tries. If they score a field goal, they will get 3 points and tie the game, forcing overtime.
   a) What are the odds in favour of each option?
   b) Which option should the coach choose?

16. Three people are running for president of the student council. The polls show that Eduard Silvestre has a 45% chance of winning, Julie Jones has a 35% chance of winning, and Bill Black has a 20% chance of winning.

a) What are the odds in favour of each person winning?

b) Suppose that Bill Black withdraws and offers his support to Julie Jones. Further suppose that his supporters also switch to Julie Jones. What are the odds in favour of Julie winning now?

prob. $\frac{11}{20} = 0.55$ to pass

$\frac{9}{20} = 0.45$ prob. to fail

17. Grant is taking a self-study course in fitness training. He must pay \$285 to take the final exam. If he fails the exam, he must pay an additional \$235 to take it again. The fitness training website lists up-to-date statistics on the pass : fail ratio. The odds that a person with good study habits will pass on his or her first try are 11 : 9. Grant can prepare for the final exam by buying three practice exams for \$65.

a) Should Grant buy the practice exams if he has good study habits? Justify your opinion. yes $\underbrace{285+65}_{\text{to do once}} < \underbrace{285+235}_{\text{to redo}}$

$\frac{17}{21} = 81\%$ chance to pass even without practice exams don't bother buying?

b) If the odds in favour of passing on the first try were 17 : 4, should Grant buy the practice exams? What if the odds in favour of passing were 3 : 7? Explain. $\frac{3}{10} = 30\%$ chance to pass – need all help can get! Buy.

$a:b$ $\quad P = \frac{a}{a+b}$ $\quad P' = \frac{b}{a+b}$

18. a) Explain why you can express the odds against an event, A, happening as $P(A'):P(A)$. ⇐ Probability

b) Suppose that the odds in favour of an event happening are $a:b$. Explain how you can determine the probability of the event happening. Give an example. $P \rightarrow \frac{a}{a+b}$

$b:a$

$P':P$

c) Suppose that the probability of an event happening is $\frac{a}{c}$. Explain how you can determine the odds against the event happening. Give an example. P of event not happening $= \frac{c-a}{c}$

Closing

odds for not happening $= c-a : a$

19. Do you prefer to express the likelihood that an event will happen using probability or odds? Explain why, and provide an example.

shows a % chance of it occuring.

Extending

20. The probability that a child between the ages of 6 and 18 will need corrective lenses to see properly is 25.4%. Of the children between the ages of 6 and 18 who do need corrective lenses, the odds in favour of them being girls are 141 : 100.

a) Determine the probability that a randomly selected girl between the ages of 6 and 18 will need corrective lenses.

b) Determine the odds that a randomly selected 18-year-old boy will need corrective lenses.

5.3

- The number of ways to seat Allison and Franco at either end is 2! or $_2P_2$.
- The number of ways to seat the other 4 friends is 4! or $_4P_4$.
- The number of ways to seat the other 12 people in the class is 12! or $_{12}P_{12}$. — must have this too since will be in the final that we will divide by.

The total number of ways to seat the 6 friends in the first row is $(_2P_2)(_4P_4)(_{12}P_{12})$.

I knew that I could multiply here, since I was seating Allison and Frank on the ends AND the other 4 friends in between AND the remaining 12 people.

The total number of ways to seat the 6 friends in any row is $3(_2P_2)(_4P_4)(_{12}P_{12})$.

Since the 6 friends can be seated in the same row 3 different ways (row 1, 2, or 3), I multiplied the number of ways they can sit in the first row by 3.

The total number of ways to assign 18 people to 18 bikes is 18! or $_{18}P_{18}$.

The number of ways that the 18 bikes can be assigned is equivalent to the number of possible permutations for a set of 18 objects.

$$P(F) = \frac{3(_2P_2)(_4P_4)(_{12}P_{12})}{_{18}P_{18}}$$

I determined the probability that the 6 friends will be in the same row by dividing the number of favourable outcomes by the total number of outcomes.

$$P(F) = \frac{3 \cdot 2! \cdot 4! \cdot 12!}{18!}$$

$$P(F) = \frac{3 \cdot 2! \cdot 4! \cdot 12!}{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12!}$$

I used the fact that $\frac{12!}{12!} = 1$ to simplify.

$$P(F) = \frac{3 \cdot 2! \cdot 4!}{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13}$$

$$P(F) = \frac{144}{13\,366\,080}$$

I used my calculator to multiply since the numbers were large.

$$P(F) = \frac{1}{92\,820}$$

The probability that all 6 friends will be in the same row, with Allison and Franco at either end, is $\frac{1}{92\,820}$.

Your Turn

Franco determined the solution to this problem by calcula

Would this calculation give the correct answer? Explain.

Pg 321 #5

a) all possibilities, no repeats
$26 \times 25 \times 10 \times 9 \times 8 = 468000$

with S + Q $2! \times 10 \times 9 \times 8 = 1440$ ✓

$\rightarrow \frac{1440}{468000} = 0.0030769$

b) all possib., no repeats allowed
$26 \times 26 \times 10 \times 10 \times 10 = 676000$

with S + Q $2! \times 10 \times 10 \times 10 = 2000$

$\rightarrow \frac{2000}{676000} = 0.00296$

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seating 4 ople.

row he y 3.

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ill be ırable

oers

/e

NEL

5.3

In General
Calculate the total # of possibilities.
Calculate the # of possibility for given situation.
Divide $\frac{\text{given}}{\text{total}}$

In Summary

Key Idea

- You may be able to use the Fundamental Counting Principle and techniques involving permutations and combinations to solve probability problems with many possible outcomes. The context of each particular problem will determine which counting techniques you will use.

Need to Know

- Use permutations when order is important in the outcomes.
- Use combinations when order is not important in the outcomes.

CHECK Your Understanding

1. A credit card company randomly generates temporary four-digit pass codes for cardholders. Suri is expecting her credit card to arrive in the mail. Determine the probability that her pass code will consist of four different even digits. 0, 2, 4, 6, 8 ↖includes zero here

all possible codes $10 \cdot 10 \cdot 10 \cdot 10 = 10000$

$5 \times 4 \times 3 \times 2 = 120$ $P = \frac{120}{10000} = 0.012$

2. In a card game called Crazy Eights, players are dealt 8 cards from a standard deck of 52 playing cards. Determine the probability that a hand will consist of 8 hearts.

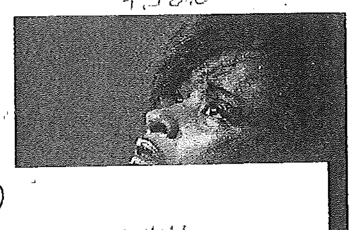
${}_{52}C_8$ = total possible hands = 7.52×

${}_{13}C_8$: hand of ♡ = 1287

$P = \frac{1287}{7.52 \times 10^8} = 0$ basically

3. From a committee of 12 people, 2 of these people are randomly chosen to be president and secretary. Determine the probability that Ben and Jen will be chosen.

all possible choices ${}_{12}P_2 = 132$
Ben + Jen chosen ${}_2P_2 = 2$
$\frac{2}{132}$



PRACTISING

pg 321 #4

4. Five boys and six girls have signed up for a trip to see l artists compete at Festival International de la Chanson Only four students will be selected to go on the trip. I probability for the following:

a) Only boys will be on the trip.

b) There will equal numbers of boys and girls on the t

c) There will be more girls than boys on the trip.

total: ${}_{11}C_4 = 330$ possibilities

a) boys only: ${}_5C_4 = 5 \rightarrow \frac{5}{330} = 0.0152$

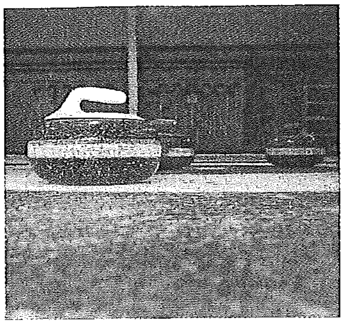
b) equal girls + boys
2 boys: ${}_5C_2$ and × 2 girls ${}_6C_2 = 150$
$\rightarrow \frac{150}{330} = 0.45$

c) 3 girls or 4 girls
${}_6C_3 \times {}_5C_1 + {}_6C_4$
$\rightarrow \frac{115}{330} = 0.348$

5. Access to a particular online game is password protecte must create a password that consists of two capital lett three digits. For each condition below, determine the a password chosen at random will contain the letters S

a) Repetitions are not allowed in a password.

b) Repetitions are allowed in a password.



6. A high-school athletics department is forming a beginners curling team to play in a social tournament. Nine students, including you and your three friends, have signed up for the four positions of skip, third, second, and lead. The positions will be filled randomly, so every student has an equal chance of being chosen for any position.
   a) Determine the probability that you and your three friends will be chosen.
   b) How would this probability change if only eight students had signed up for the team?

7. There are nine players on a baseball team, all with roughly equal athletic ability. The coach has decided to choose the players who will play the four infield positions (first base, second base, third base, and shortstop) randomly. Tara and Laura are on the team. Determine the odds in favour of Tara and Laura being chosen to play in the infield.

8. A student council has 15 members, including Yuko, Luigi, and Justin.
   a) The staff advisor will select three members at random to be treasurer, secretary, and liaison to the principal. Determine the probability that the staff advisor will select Yuko to be treasurer, Luigi to be secretary, and Justin to be liaison. *order matters*
   b) The staff advisor will also select three members at random to clean up after the pep rally. Determine the probability that the staff advisor will select Yuko, Luigi, and Justin to do this. *order doesn't matter*

a) all possibilities
$_{15}P_3 = \frac{15!}{12!} = 2730$
specific Y as T, L as S, J as L
$1 \times 1 \times 1 = 1$
$\rightarrow \frac{1}{2730} = 0.0003663$

b) all possibilities
$_{15}C_3 = \frac{15!}{3!12!} = 455$
Y, L and J selectted $= {}_3C_3 = 1$
$\rightarrow \frac{1}{455} = 0.0022$

9. Lesley needs to create a four-digit password to access her voice mail. She can repeat some of the digits, but all four digits cannot be the same.
   a) Determine the probability that her password will be greater than 5000.
   b) Determine the probability that the first and last digits of her password will be 4.
   c) Determine the probability that the first digit of her password will be odd and the last digit will be even.

10. A student council consists of 16 girls and 7 boys. To form a subcommittee, 5 students are randomly selected from the council. Determine the odds in favour of 3 girls and 2 boys being on the subcommittee.

11. Larysa tosses four coins. Determine the probability that at least one coin will land as tails. $\frac{\text{all posibilities} - \text{all heads}}{\text{all posibilities}} = \frac{2\times2\times2\times2 - 1}{2\times2\times2\times2} = \frac{15}{16}$

12. Five friends, including Bilyana and Bojana, are sitting in a row in a theatre.
   a) Determine the probability that Bilyana and Bojana are sitting together.
   b) Determine the probability that they are not sitting together.

13. Tanya is planning her schedule for university. She wants to take the following courses during her first two terms: biology, English, psychology, religion, linear algebra, political studies, economics, and philosophy. She is equally likely to take any of these courses in either term, since they are all introductory courses.
   a) Suppose that Tanya decides to take four courses in her first term. Determine the probability that three of them will be psychology, linear algebra, and English.
   b) Suppose that Tanya decides to take five courses in her first term. Determine the probability that three of them will be religion, political studies, and biology.

14. Doc deals you eight cards at random from a standard deck of 52 playing cards. Determine the probability that you have the following hands.
   a) A, 2, 3, 4, 5, 6, 7, 8 of the same suit
   b) Eight cards of the same colour
   c) Four face cards and four other cards

15. Erynn has letter tiles that spell CABINET. She has selected three of these tiles at random. Determine the probability that the tiles she selected are two vowels and one consonant.

all possible # of selections of 3 tiles

✓ ${}_7C_3 = \frac{7!}{3!4!} = 35$

2 vowels, 1 consonant

${}_3C_2 \times {}_4C_1 = \frac{3!}{2!1!} \times \frac{4!}{1!3!} = 12$

$\rightarrow \frac{12}{35} = 0.343$

16. At a local dog show, dogs compete in eight different categories. The eight winners of these categories are all different breeds, including a sheltie and a bearded collie. The organizers randomly line up the eight winners for the "Best in Show" competition. Determine the probability that the bearded collie and the sheltie will be next to each other in the lineup. (somewhere in lineup)

no preference of order: $8! = 40320 = {}_8P_8$

$\rightarrow 2! \times 7! = 10080$

C+S

$\rightarrow \frac{10080}{40320} = 0.25$

17. The starting lineup for a basketball team consists of two guards and three forwards. On the team that sisters Maggie and Tanya play for, there are seven forwards and five guards from which the coach can choose a starting lineup. Maggie is a guard and Tanya is a forward. For the first exhibition game of the year, the coach will select the starting players at random. What is the probability that both Maggie and Tanya will be in the starting lineup?

Closing

18. Explain when you would use permutations to solve a probability problem and when you would use combinations. Give an example.

pg 327 Mid-Chapter Review

$4 \times 4 \times 4 = 64$ possible outcomes

5.1

1a) ← $\boxed{\text{omit 1a}}$

	1				2				3				4			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4
sum	3 4 5 6	4 5 6 7	5 6 7 8	6 7 8 9	4 5 6 7	5 6 7 8	6 7 8 9	7 8 9 10	5 6 7 8	6 7	7		6 7	7		
	1 2 3 4	2 4 6 8	3 6 9 12	4 8 12 16	2 [illegible] 12 4	4 8 12 16	7 ___	8 ___	3 6 9 _	6 12 __	9 __	____	4 8 __	8 __	___	___

Product wins in $\frac{48}{64}$ cases

Product bigger more often so Ethan wins ✓

b)

h < h → hh → Bob
h < T → ~~hT~~
T < h → ~~Th~~
T < T → TT → Anne

fair game ✓

	H	T
H	HH	HT
T	HT	TT

2. a)

sum	1	2	3	4	5
1	2	3	4	5	6
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9

b) i) $\frac{4}{20} = 0.2 = 20\%$ of the time

ii) $\frac{3}{20} = 0.15 = 15\%$ ✓

iii) $\frac{3}{20} = 0.15 = 15\%$

5.2

3. a) $\frac{26}{52} = 0.5$

b) 26:26 → 1:1

c) ♡ Clubs spade ◇
39:13 → 3:1 ✓

d) JQK ×4 = 12 → 40 not face cards $\frac{40}{52} = 0.769$

4. Odds for him winning → 5:23 (win lose)

prob for winning $= \frac{5}{28} = 0.179$ ✓

5a) Prob. bit < 50% $= \frac{50}{100}$ ⇒ odds bit less than 50:100−50 → 1:1 (for against)

b) bit more than 1:1

c) yes John A McDonald, Jean Chrétien − Jan 11
1815 1934

§5.3 Mid-ch Review

6. all possible choices $_{14}C_3 = \frac{14!}{3!11!} = 364$ or $_{14}P_3 = \frac{14!}{11!} = 2184$

F S + A chosen $_3C_3 = \frac{3!}{3!0!} = 1$ $_3P_3 = \frac{3!}{0!} = 6$

prob: ⟶ $\frac{1}{364} = 0.00275 = \frac{6}{2184} = \frac{1}{364}$

works

better since diff position

✓

7. A, 9, 10, J, Q, K ×4 5-card hands → total # of possibilities = $_{24}C_5 = 42504$ ✓

a) AKQJ of same suit ⟶ $4 \times {_{20}C_1} = 80$ → $P = \frac{80}{42504} = 0.00188$ → 0.19%

4 hand possibilities since 4 suits; 20 cards remaining to choose from

b) 5 cards same colour $2 \times {_{12}C_5} = 2 \times \frac{12!}{5!7!} = 1584$ ⟶ $P = \frac{1584}{42504} = 0.037$

2 possibilities: black or red

c) 4 of a kind (same card) $(6 \times {_4C_4}) \times {_{20}C_1} = 120$ ⟶ $P = \frac{120}{42504} = 0.0028$ ✓

6 diff types of cards; 1; 5th card 20

8. # of possible orders of playlist ⟶ $30! = 2.65252 \times 10^{32}$ $\frac{6!\,25!}{30!}$

# of ways these 6 song can play tog. in a playlist ⟶ $6! \times 25! = 1.1168 \times 10^{28}$

6! fall; 25! rest plus (counting 6 tog as 1)

⟶ $P = \frac{1.1168 \times 10^{28}}{2.65252 \times 10^{32}} = 0.00004$ ✓

$\underbrace{(6) \; \_\_ \; 24}_{25}$

9. total $_{12}C_3 = \frac{12!}{3!9!} = 220$ possible groups of 3 coins dropped

specific $_6C_2 \times {_6C_1} = \frac{6!}{2!4!} \cdot \frac{6!}{1!5!} = 90$

2 loonie 1 other

$P = \frac{90}{220} = 0.409$ ✓