Measurement of Motion Extra Practice Worksheet Name: Key

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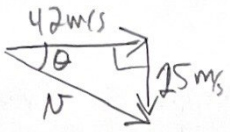
25 ✓

1. An airplane is aiming east at 42m/s, but a wind is blowing from the north at ~~25~~m/s. What is the airplane's velocity over the ground?



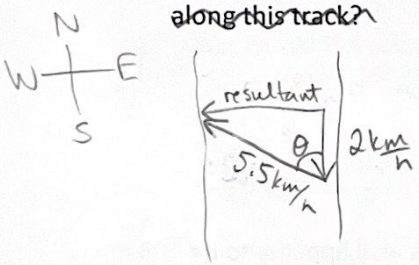
$v^2 = 42^2 + 25^2$

$v = 49\ m/s$

$\tan\theta = \frac{25}{42}$ $\theta = 31°$

$\vec{v} = 49\ m/s\ [31°\ S\ of\ E]$

2. A river flows south at 2.0km/h. A boat that moves at 5.5km/h in still water, wants to cross to the west side. What direction should the boat aim ~~and how fast will it be moving along this track?~~ so it travels straight across



aim + current = resultant

5.5 + 2 = ... but not linear

$\cos\theta = \frac{2}{5.5}$ $\theta = 69°$ W of N

or

21° N of W

3. An astronaut is travelling at a speed of 0.785c relative to Earth. The astronaut's timing device indicates that the trip takes 2.45 years. How long would an observer on Earth see the trip as taking?

v

t_0 $t = ?$

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2.45\ y}{\sqrt{1 - \frac{(0.785c)^2}{c^2}}} = \frac{2.45\ y}{0.619496} = 3.95\ yrs$$

4. You are travelling through space at a speed of 2.8×10^8 m/s relative to Earth. If your friend, who stayed back on Earth, aged 15 years during your trip, how many years did you age?

$t_o = ?$ $\quad t = 15\text{y}$

$$t_o = t\sqrt{1-\frac{v^2}{c^2}} = 15\text{y}\sqrt{1-\frac{(2.8\times10^8\text{ m/s})^2}{(3\times10^8\text{ m/s})^2}} = 5.4\text{ yrs}$$

5. An observer on Earth measures a spacecraft's trip to take 6.5 years. On the spacecraft, the timing device only measures 1.2 years. How fast was the spacecraft travelling? (Show your rearrangement of the formula.)

$t = 6.5$ years $\quad t_o = 1.2$ years

$$\left(\sqrt{1-\frac{v^2}{c^2}}\right)^2 = \left(\frac{t_o}{t}\right)^2$$

$$1-\frac{v^2}{c^2} = \frac{t_o^2}{t^2}$$

$$1-\frac{t_o^2}{t^2} = \frac{v^2}{c^2}$$

$$\sqrt{c^2\left(1-\frac{t_o^2}{t^2}\right)} = \sqrt{v^2}$$

$$v = c\sqrt{1-\frac{t_o^2}{t^2}}$$

$$= c\sqrt{1-\frac{1.2^2}{6.5^2}}$$

$$= 0.9659c = 0.97c$$

$$= 2.9\times10^8\text{ m/s}$$

6. A spaceship is travelling at 0.87c parallel to a brick wall. If this wall appears to be 103 m long to the astronaut in the spaceship, how long would an observer standing next to the wall see it as?

$v = 0.87c$ $\quad L = 103\text{ m}$ $\quad L_o = ?$

$$L_o = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{103\text{ m}}{\sqrt{1-\frac{(0.87c)^2}{c^2}}} = \frac{103\text{ m}}{0.49305} = 210\text{ m}$$

7. A spacecraft is travelling to Alpha Centauri, which is 4.2 light years away from Earth. The astronauts measure the distance as only 2.1 light years. At what speed must the spacecraft be travelling? (Show your rearrangement of the formula.)

$L_o = 4.2$ light years $\quad L = 2.1$ light years

$$\left(\frac{L}{L_o}\right)^2 = \left(\sqrt{1-\frac{v^2}{c^2}}\right)^2$$

$$\frac{L^2}{L_o^2} = 1-\frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = 1-\frac{L^2}{L_o^2}$$

$$v^2 = c^2\left(1-\frac{L^2}{L_o^2}\right)$$

$$v = c\sqrt{1-\frac{L^2}{L_o^2}}$$

$$= c\sqrt{1-\frac{2.1^2}{4.2^2}}$$

$$= 0.866025c$$

$$= 0.87c$$

$$= 2.6\times10^8\text{ m/s}$$

8. What is the length and height of a brick wall as seen by a vehicle moving along the ground at 2.84×10^8? The wall was built to be 3.50 m tall and 86.7 m long.

L — remains the same; only dimension parallel to motion changes

v L_o

$$L = L_o\sqrt{1-\frac{v^2}{c^2}}$$

$$= 86.7\text{m}\sqrt{1-\frac{(2.84\times10^8)^2}{(3\times10^8)^2}} = 27.9\text{m}$$

9. How much energy would be produced from a medium sized orange (154g) if there were a way to convert the entire thing into energy?

0.154kg

$$E = mc^2$$

$$= (0.154\text{kg})(3\times10^8\,\text{m/s})^2 = 1.39\times10^{16}\,\text{J}$$

10. What would the relativistic mass of an electron be if it is moving at 2.9×10^8 m/s?

v

$m = ?$ $m_o = 9.11\times10^{-31}\text{kg}$

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{9.11\times10^{-31}\text{kg}}{\sqrt{1-\frac{(2.9\times10^8\,\text{m/s})^2}{(3\times10^8\,\text{m/s})^2}}} = \frac{9.11\times10^{-31}\text{kg}}{0.256038} = 3.6\times10^{-30}\text{kg}$$

11. If a proton has a relativistic mass of 2.56×10^{-26} kg, how fast is it moving? (Show your rearrangement of the formula.)

m $m_o = 1.67\times10^{-27}\text{kg}$

$$\left(\sqrt{1-\frac{v^2}{c^2}}\right)^2 = \left(\frac{m_o}{m}\right)^2$$

$$1-\frac{v^2}{c^2} = \frac{m_o^2}{m^2}$$

$$1-\frac{m_o^2}{m^2} = \frac{v^2}{c^2}$$

$$\sqrt{c^2\left(1-\frac{m_o^2}{m^2}\right)} = \sqrt{v^2}$$

$$v = c\sqrt{1-\frac{m_o^2}{m^2}}$$

$$= c\sqrt{1-\frac{(1.67\times10^{-27})^2}{(2.56\times10^{-26})^2}}$$

$$= 0.99787c$$

$$= 0.998c$$

$$= 2.99\times10^8\,\frac{\text{m}}{\text{s}}$$

$m_0 = ?$ m

12. What is the rest mass of an object with a relativistic mass of 563 kg if it is travelling at 0.93c?

v

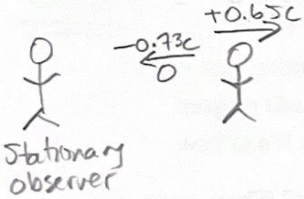
$$m_0 = m\sqrt{1-\frac{v^2}{c^2}} = 563\text{kg}\sqrt{1-\frac{(0.93c)^2}{c^2}}$$

$$= 207\text{kg}$$

13. A spacecraft travelling away from Earth at a velocity of 0.94c emits a flash of light. What is the velocity of the light flash as seen by an observer on Earth?

$$u = \frac{v+u'}{1+\frac{vu'}{c^2}} = \frac{0.94c + c}{1+\frac{(0.94c)(c)}{c^2}} = c$$

14. A person moving at 0.65c away from a stationary observer throws an object backward at a velocity of 0.73c (relative to the moving person). What is the velocity of the object with respect to the stationary observer?

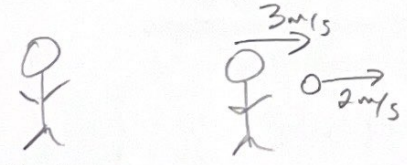


$$u = \frac{v+u'}{1+\frac{vu'}{c^2}} = \frac{-0.73c + 0.65c}{1+\frac{(-0.73c)(0.65c)}{c^2}}$$

$$= \frac{-0.08c}{0.5255} = -0.15c$$

$$= 0.15c \text{ toward observer}$$

15. A person moving at 3.0 m/s away from a stationary observer throws an object forward at a velocity of 2.0 m/s (relative to the moving person). What is the velocity of the object with respect to the stationary observer?



$$u = \frac{v+u'}{1+\frac{vu'}{c^2}}$$

$$= \frac{3+2}{1+\frac{3(2)}{c^2}} = 5.0\text{ m/s}$$

$\frac{3(2)}{c^2} \approx 0$