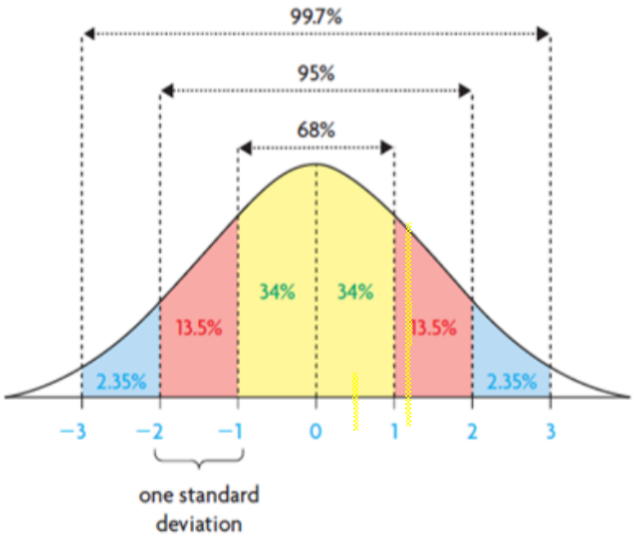
5.5 Z-Scores

Z- Scores [5.5]

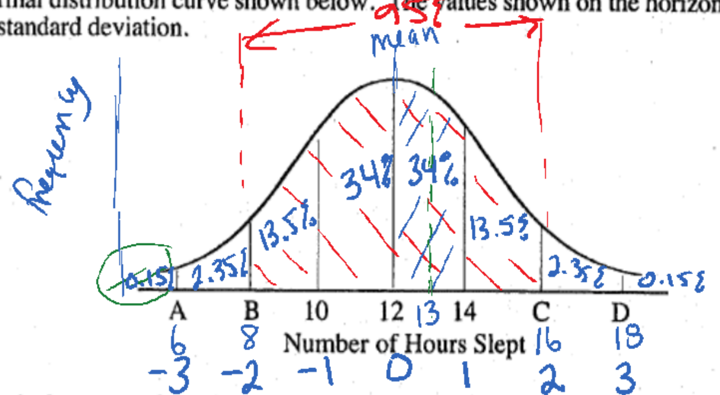
In Summary

Key Ideas

- The standard normal distribution is a normal distribution with mean, μ, of 0 and a standard deviation, σ, of 1. The area under the curve of a normal distribution is 1.
- *Z*-scores can be used to compare data from different normally distributed sets by converting their distributions to the standard normal distribution.



A nurse records the number of hours an infant sleeps during a day. He then records the data on a normal distribution curve shown below. The values shown on the horizontal axis differ by one standard deviation.



a) What is the mean of the data?

12 hours

b) What is the standard deviation?

2 hrs

c) What are the values for A, B, C, and D?

6, 8, 16, 18

d) What percentage of a day, to the nearest hundredth, does the infant sleep:

i) between 12 and 14 h?

34%

ii) between 8 and 16 h?

95%

13.5+34+34+13.5

iii) less than 6 h?

0.15%

e) Why is it not possible at this time to determine the percentage of a day that the infant sleeps for less than 13 hours?

This model is limited to whole standard deviations ... for ½ deviations we need Z-scores !!

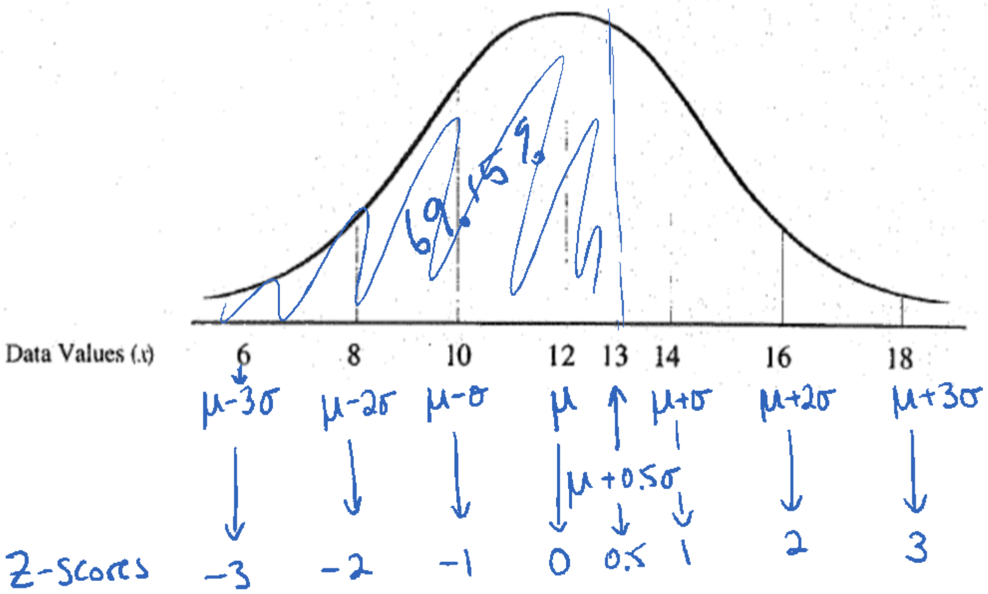
z–scores

A ***z*-score** for a data value describes the number of standard deviations above or below the mean.

Displaying z-scores on the Normal Curve

Consider Class Example #1 with a mean value of 12 and a standard deviation of 2.

- The data value 16 is 2 standard deviations above the mean and has a *z*-score of 2.
- The data value 6 is 3 standard deviations below the mean and has a *z*-score of -3.
- The data value 12 is 0 standard deviations away from the mean and has a *z*-score of 0.
- The data value 13 is 0.5 standard deviations above the mean and has a *z*-score of 0.5.



z-score *Formula*

z-scores can be calculated using the formula

$$z = \frac{x - \mu}{\sigma}$$

where,
z is the z-score
x is the particular data value
μ is the mean (average)
σ is the standard deviation

This formula is on the formula sheet

from example $z_{13\text{ hours}} = \frac{13-12}{2} = 0.5$ z-score on graph

Example 2: Determine at which location Hailey's run time was better when compared to club results.

Hailey and Serge belong to a running club in Vancouver. Part of their training involves a 200 m sprint. Below are normally distributed times for the 200 m sprint in Vancouver and on a recent trip to Lake Louise. At higher altitudes, run times improve.

Location	Altitude (m)	Club Mean Time: μ for 200 m (s)	Club Standard Deviation: σ (s)	Hailey's Run Time (s)	Serge's Run Time (s)
Vancouver	4	25.75	0.62	24.95	25.45
Lake Louise	1661	25.57	0.60	24.77	26.24

Tony's midterm marks are shown below, together with the class mean and standard deviation for each subject. By calculating z-scores, determine in which subject Tony performed best relative to the rest of the class.

$z = \frac{x-\mu}{\sigma}$

Subject	Tony's Mark (x)	Mean Mark (μ)	Standard Deviation (σ)
Mathematics	74	68	12
Chemistry	79	73	14
Physics	68	66	11

Math

$z = \frac{74-68}{12}$

$= 0.50$

Chem

$z = \frac{79-73}{14}$

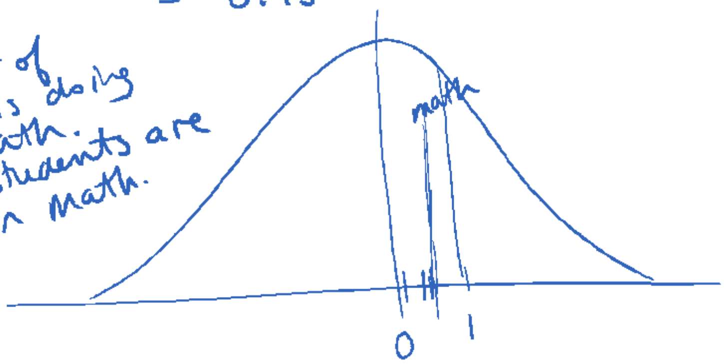
$= 0.43$

Phys

$z = \frac{68-66}{11}$

$= 0.18$

Relative the rest of the class, he is doing better in math. more of the students are below him in math.



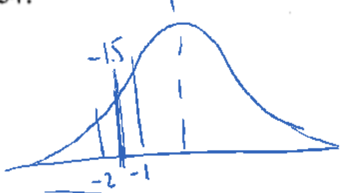
pg22

The marks on a math exam at a university were found to have a mean of 52 with a standard deviation of 12. A professor who thought the exam was too difficult, decided to adjust the original marks by raising the mean to 65, while reducing the standard deviation to 10 and leaving the z-scores unchanged. What would the new mark be for a student who received an original mark of 34?

original results

$\mu_1 = 52$
$\sigma_1 = 12$
z_1
$x_1 = 34$

$\rightarrow z$



adjusted results

$\mu_2 = 65$
$\sigma_2 = 10$
$z_1 = -1.5 = z_2$
$x_2 = ?$

$z_1 = \frac{x_1 - \mu_1}{\sigma_1}$

$= \frac{34-52}{12}$

$= -1.5$

$z_2 = \frac{x_2 - \mu_2}{\sigma_2}$

$10 \times (-1.5) = \left(\frac{x_2 - 65}{\cancel{10}}\right)\cancel{10}$

$-15 + 65 = x_2 - 65 + 65$

$50 = x_2$

Example 5:

IQ tests are sometimes used to measure a person's intellectual capacity at a particular time. IQ scores are normally distributed, with a mean of 100 and a standard deviation of 15. If a person scores 119 on an IQ test, how does this score compare with the scores of the general population?

$x = 119$
$\sigma = 15$
$\mu = 100$

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{119 - 100}{15} = 1.2666 = 1.27$$

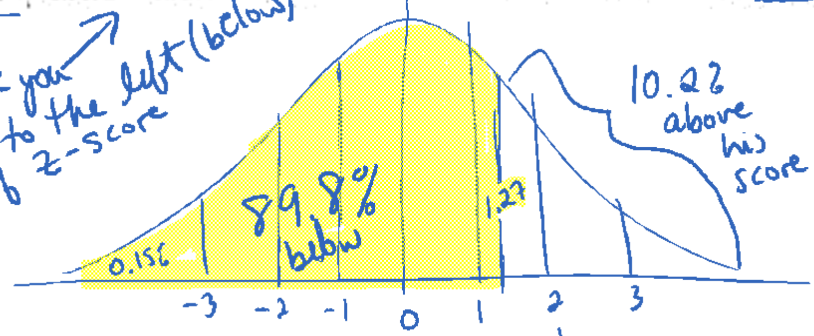
use z-score table to see the percent that corresponds to $z = 1.27$

$1.2 + 0.07$ ← across top
↑ left column ⟹ $0.8980 = 89.8\%$

∴ 119 score on IQ test means you are above the 89.8% of the population

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319

give you % to the left (below) of z-score



to find above/right subtract from 100.

$\mu - \sigma$ μ $\mu + \sigma$ $\mu + 2\sigma$

Practice pg 264
#5ac, 6ac, 7a, 8a, 9, 10

Pg 24

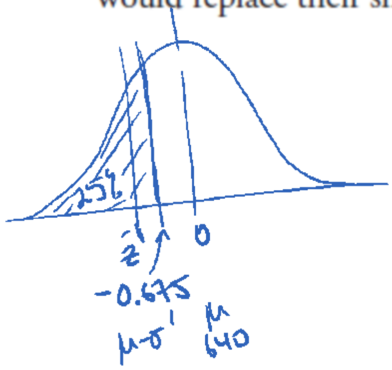
Example 6:

$$z=\frac{x-\mu}{\sigma}$$

Athletes should replace their running shoes before the shoes lose their ability to absorb shock.

Running shoes lose their shock-absorption after a mean distance of 640 km, with a standard deviation of 160 km. Zack is an elite runner and wants to replace his shoes at a distance when only 25% of people would replace their shoes. At what distance should he replace his shoes?

$\mu = 640$ km
$\sigma = 160$ km



25% $\xrightarrow[\text{table}]{\text{use}}$ between −0.67 and −0.68
→ −0.675

$z = -0.675$

How many km does $z = -0.675$ equate to?

$$z=\frac{x-\mu}{\sigma}$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514 25%	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776

$$160(-0.675) = \left(\frac{x-640}{\cancel{160}}\right) \times \cancel{160}$$

$$160(0.675) + 640 = x$$

$$x = 532 \text{ km}$$

Example 7:

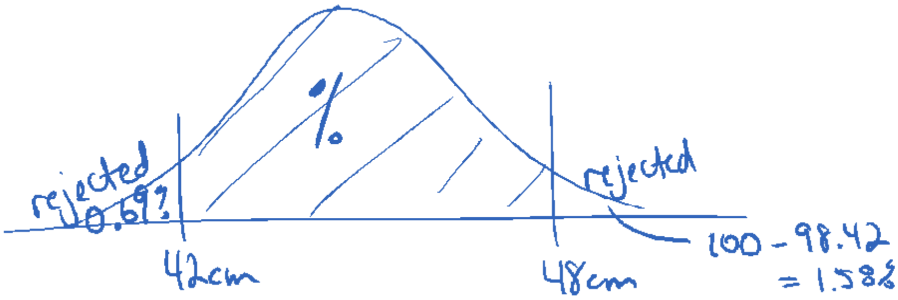
$\mu = 45.2\text{cm}$
$\sigma = 1.3\text{cm}$

The ABC Company produces bungee cords. When the manufacturing process is running well, the lengths of the bungee cords produced are normally distributed, with a mean of 45.2 cm and a standard deviation of 1.3 cm. Bungee cords that are shorter than 42.0 (x_1) cm or longer than 48.0 (x_2) cm are rejected by the quality control workers.

a) If 20 000 bungee cords are manufactured each day, how many bungee cords would you expect the quality control workers to reject?

b) What action might the company take as a result of these findings?

① get a percent from z-score table but need z-score from formula



$z = \frac{42-45.2}{1.3}$ $= -2.4615$

from z-score table → 0.69%

$z = \frac{48-45.2}{1.3}$ $= 2.1538$ → 98.42%

reject $0.69 + 1.58 = 2.27\%$

∴ how many bungee cords?

$0.0227 \times 20\,000$ bungee cords

$= 454$ bungee cords rejected

Pg 264 # 11, 13, 16, 19 (21 - mentally / share with a partner)