Lesson 3 ELECTRIC POTENTIAL DUE TO A POINT

NOTES

If a charged object is in an electric field, it has electric potential energy ($E_{p(e)}$); in other words, work was done on the charged object to move it into its current position, and it now has the potential to move in that field.

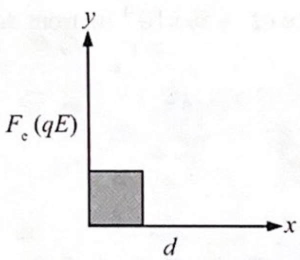
Earlier, we discussed gravitational potential energy in a non-uniform gravitational field. The equation that described this gravitational potential energy is

$$E_{p(g)} = -\frac{Gm_1m_2}{r}$$

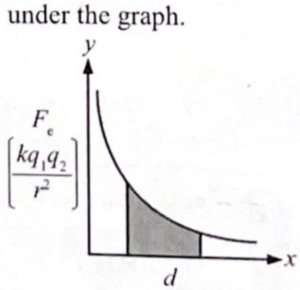
Recall that calculus was necessary to derive this formula.

We encounter the same problem again. In a uniform electric field (such as exists between charged parallel plates), work done in moving a charged object placed in this uniform field is equal to the electrical potential energy gained by this charged object, and this is equal to the area under the graph below (the area of a rectangle).

Work done against an electric force = $F_e d = qEd$



In the non-uniform electric field caused by a point charge, the electric potential energy can not be found as easily, but the reasoning is the same. Work done in moving a charged object in this non-uniform field is equal to the electrical potential energy gained and this is still the same as the area under the graph.



Using calculus, this area (the electrical potential energy) can be determined to be

$$E_{p(e)} = \frac{kq_1q}{r}$$

NOTES

Electric potential (V) is defined as the electric potential energy per unit charge.

$$V = \frac{E_{p(e)}}{q_t}$$ (q = charge of test object)

$\frac{\text{Energy}}{\text{charge}} = V$ ← scalar

From this definition, we can derive the equation

$$V = \frac{kq_s}{r}$$ (q_1 = charge of object producing the potential)

- so use signs of charges in equations
- and no direction to worry about
- (no gravitational equivalent $\frac{E}{m}$??)

Derivation:

$$V = \frac{E_{p(e)}}{q}$$

$$V = \frac{\frac{kq_1 q}{r}}{q}$$

or

$$V = \frac{kq_1}{r}$$

The test charge cancels out, so we are left with the charge of the object producing the electric potential.

Note: The electric potential at a point is defined in terms of the moving of a positive charge. Therefore, V can be positive or negative.

We sometimes want to find the electric potential between two points. The electric potential between two points is the potential difference.

i.e.,Given two points, A and B, the potential difference between A and B is:

V_{AB}	=	V_B	−	V_A
Potential difference between points A and B		Electric potential at B		Electric potential at A

$\Delta V_{AB} = V_B - V_A$

$W = \Delta E_p = \Delta V q_t$

$\Delta E_k = \Delta E_p$

When we talk of the potential at a point, we are actually talking about the potential difference between that point and infinity. The potential at infinity is assigned a value of zero.

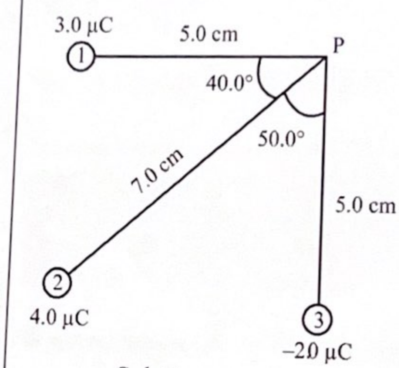
NOTES

When the electric force works on a test charge, the kinetic energy increases and the potential energy decreases. If the potential energies between two points A and B are $E_{p(eA)}$ and $E_{p(eB)}$, then the difference in potential energy denotes the negative work done.

$$V_{BA} = V_B - V_A = \frac{\left(E_{p(eB)} - E_{p(eA)}\right)}{q} = -\frac{W_{BA}}{q}$$

Example

Calculate the potential at point P given the following diagram:



Solution

Find the potential due to each charge

$$V_1 = \frac{kq_1}{r_1}$$

$$= \frac{\left(9.00\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(3.0\times10^{-6}\ \text{C})}{5.0\times10^{-2}}$$

$$= 5.40\times10^5\ \text{V}$$

$$V_2 = \frac{kq_2}{r_2}$$

$$= \frac{\left(9.00\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(4.0\times10^{-6}\ \text{C})}{7.0\times10^{-2}\ \text{m}}$$

$$= 5.14\times10^5\ \text{V}$$

$$V_3 = \frac{kq_3}{r_3}$$

$$= \frac{\left(9.00\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(-2.0\times10^{-6}\ \text{C})}{5.0\times10^{-2}\ \text{m}}$$

$$= 3.60\times10^5\ \text{V}$$

include sign & charge since scalar

Fixed V

neg.

NOTES

(potentials due to negative charges are negative)

Add potentials (Note: potentials are scalars)

$V_T = V_1 + V_2 + V_3$

$= (5.40 \times 10^5 \text{ V}) + (5.14 \times 10^5 \text{ V}) + (-3.60 \times 10^5 \text{ V})$

$= 6.9 \times 10^5 \text{ V}$

Example 2

How much work is done against an electric field produced by a 2.5 μC ← source charged object when an 0.025 μC charge is moved from $r = 3.0$ cm to $r = 1.0$ cm?

Solution

Find potentials at each point

$V_i = \dfrac{kq_s}{r_i}$

$= \dfrac{\left(9.00 \times 10^9 \dfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(2.5 \times 10^{-6} \text{ C})}{(3.0 \times 10^{-2} \text{ m})}$ ← source charge

$= 7.50 \times 10^5 \text{ V}$

$V_f = \dfrac{kq_1}{r_f}$

$= \dfrac{\left(9.00 \times 10^9 \dfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(2.5 \times 10^{-6} \text{ C})}{(~~3.0 \times 10^{-2}~~ \text{ m})}$ 0.01m

~~$= 7.50 \times 10^5$~~ V $= 2.25 \times 10^6$ V ⇐ fixed

$W = q_t(V_f - V_i)$

$= (2.5 \times 10^{-8} \text{ C})(2.25 \times 10^6 \text{ V} - 7.5 \times 10^5 \text{ V})$

$= 3.8 \times 10^{-2}$ J ← test

if $V_f - V_i < 0$, just make it positive, especially if finding v by taking square root

$\frac{1}{2}mv^2 = \frac{kq_sq_t}{r_f} - \frac{kq_sq_t}{r_i}$

(#4 and #7 in problems)

~~at least #1, 5, 2, 4, 6, 8~~ (All)

$E_p = \frac{k q_s q_t}{r}$

PRACTICE EXERCISES

Formula: $V = \frac{kq_s}{r}$ (source) $V = \frac{E_p}{q_t}$ (test) $\Delta E_k = \Delta E_p$ $W = \Delta E_p = \Delta V q_t$

1. What is the potential at a distance of 6.0 cm from a 2.5 μC charge?

$V = \frac{kq}{r} = \frac{9\times10^9(2.5\times10^{-6})}{0.06\text{ m}}$

$= 3.8\times10^5\text{ V}$ ✓

2. What is the potential at a distance of 25 cm from a –2.5 μC charge?

$V = \frac{k(-2.5\times10^{-6})}{(0.25\text{ m})} = -9.0\times10^4\text{ V}$ ✓

3. How much work is done against the electric field produced by a 5.0 μC (q_s) charged object when a (q_t =) 0.030 μC charge is moved from r = 45 cm to r = 15 cm?

$W = \Delta V q_t = \left(\frac{kq_s}{r_f} - \frac{kq_s}{r_i}\right) q_t$

$= kq_s q_t \left(\frac{1}{r_f} - \frac{1}{r_i}\right)$

$= 9\times10^9(5\times10^{-6})(0.03\times10^{-6})\left(\frac{1}{0.15} - \frac{1}{0.45}\right) = 0.006$

$= 6.0\times10^{-3}\text{ J}$ ✓

4. A proton (q_t) is released 2.0×10^{-11} m from the centre of a 6.4×10^{-18} C fixed charged sphere. What is the speed of this proton when it is 0.50 m from this centre?

(must use absolute value for ΔE_p)

$|\Delta E_p| = \Delta E_k$

$|q_t(V_f - V_i)| = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ ($v_i = 0$)

$\left|q_t\left(\frac{kq_s}{r_f} - \frac{kq_s}{r_i}\right)\right| = \frac{1}{2}mv_f^2$

$\left|1.6\times10^{-19}\left(\frac{9\times10^9(6.4\times10^{-18})}{0.5} - \frac{9\times10^9(6.4\times10^{-18})}{2\times10^{-11}}\right)\right|$

$\left|1.6\times10^{-19}(1.15\times10^{-7} - 2880)\right|$

$\Delta E = 4.608\times10^{-16}$

$= \frac{1}{2}mv^2$

$v = \sqrt{\frac{2(4.608\times10^{-16})}{1.67\times10^{-27}}}$

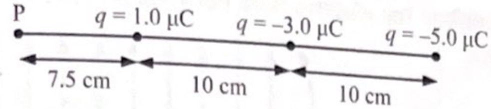
$= 7.4\times10^5\text{ m/s}$ ✓

(+) p p
$E_p = E_p + E_k$

5. Three charges are located on a line as shown below:

scalar so just find all V_s and add together

scalar use signs in equation



Find the potential at point P.

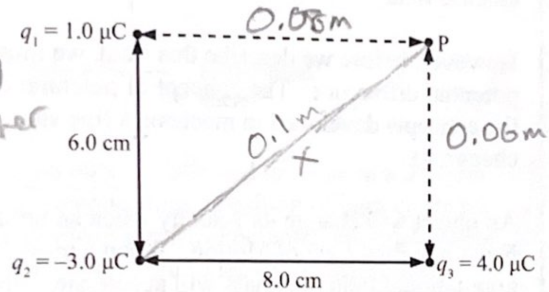
$V = \frac{Kq_s}{r}$

$V_{total} = \frac{K(1\times10^{-6})}{0.075m} + \frac{K(-3\times10^{-6})}{0.175m} + \frac{K(-5\times10^{-6})}{0.275m} = -2.0\times10^5 V$ ✓

6. Three charges are located at the corners of a rectangle as shown below:

find potential due to each charge and add together

$V = \frac{Kq_s}{r}$



$x = \sqrt{(0.08)^2 + (0.06)^2}$
$= 0.1m$

Find the potential at point P.

$V_{total} = \frac{K(1\times10^{-6})}{0.08} + \frac{K(-3\times10^{-6})}{0.1} + \frac{K(4\times10^{-6})}{0.06} = 4.4\times10^5 V$ ✓

start end
$E_p = E_p + E_k + E_k$

7. √The centres of two alpha particles are held 2.5×10^{-12} m apart, then they are released. Calculate the speed of each alpha particle when they are 0.75 m apart.

2 moving apart

$V_i = 0$
$r_f = 0.75m$
$r_i = 2.5\times10^{-12}m$

$\Delta E_k = \Delta E_p$

$\frac{1}{2}mv_f^2 + \frac{1}{2}mv_f^2 = Kq_sq_t\left|\left(\frac{1}{r_f} - \frac{1}{r_i}\right)\right|$

$v = \sqrt{\frac{(9\times10^9)(3.2\times10^{-19})^2}{6.65\times10^{-27}kg}\left|\left(\frac{1}{0.75} - \frac{1}{2.5\times10^{-12}}\right)\right|} = 2.3\times10^5 m/s$ ✓

8. √In moving a 3.00 μC charge at a constant speed from point A to point B, 4.40×10^{-5} J of work is done. If A and B are 2.4 cm apart, what is the potential difference between A and B?

irrelevant

$W = 4.4\times10^{-5} J$
$q_t = 3\times10^{-6}$
$\Delta r = 0.024 m$
$V = ?$

$W = \Delta V q_t$

$\Delta V = \frac{W}{q_t} = \frac{4.4\times10^{-5}J}{3\times10^{-6}C} = 14.7V$ ✓