Measurement of Motion Review for 2/3-term Name: Key

1. Time dilation $t = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}}$

a) A spaceship is moving at 0.80c relative to Earth. How much time elapses on the spaceship's $? = t_o$ clock for every 1.0 year that passes on Earth?

$$t_o = t\sqrt{1-\frac{v^2}{c^2}} = 1yr\sqrt{1-\frac{(0.8\cancel{c})^2}{\cancel{c^2}}} = \underline{0.60\ yrs}$$

b) An observer on Earth sees a clock on a spaceship moving at 0.60c. How much time passes on Earth for every 1.0 second that is measured by the spaceship clock?

t_o $t=?$

$$t = \frac{1s}{\sqrt{1-\frac{(0.6\cancel{c})^2}{\cancel{c^2}}}} = \frac{1s}{0.8} = \underline{1.3s}$$

2. Length contraction $L = L_o\sqrt{1-\frac{v^2}{c^2}}$

a) A spaceship is 100. m long when at rest. How long is the spaceship when it is moving at 0.90c?

L_o $L=?$

$$L = 100m\sqrt{1-\frac{(0.9\cancel{c})^2}{\cancel{c^2}}} = \underline{44m}$$

b) A pole appears to be 5.0 m long when it is moving at 0.50c, as measured by an observer on Earth. What is the rest length of the pole?

L $L_o = ?$

$$L_o = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{5\ m}{\sqrt{1-\frac{(0.5\cancel{c})^2}{\cancel{c^2}}}} = \frac{5}{0.866025} = \underline{5.8m}$$

3. Mass increase $m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$

a) A proton has a rest mass of 1.67×10^{-27} kg. What is the proton's mass when it is moving at 0.700c?

m $m=?$

$$m = \frac{1.67\times10^{-27}kg}{\sqrt{1-\frac{(0.7\cancel{c})^2}{\cancel{c^2}}}} = \frac{1.67\times10^{-27}kg}{0.71414} = \underline{2.3\times10^{-27}kg}$$

b) An electron has a rest mass of 9.11×10^{-31} kg. What is the speed of the electron when its mass is twice its rest mass?

m_o $2m_o$

$$1-\frac{v^2}{c^2} = \frac{m_o^2}{m^2}$$

$$-\frac{v^2}{c^2} = \frac{m_o^2}{m^2} - 1$$

$$\sqrt{v^2} = \sqrt{c^2\left(1-\frac{m_o^2}{m^2}\right)}$$

$$v = c\sqrt{1-\frac{(9.11\times10^{-31})^2}{(2\times9.11\times10^{-31})^2}} = \underline{0.866c}$$

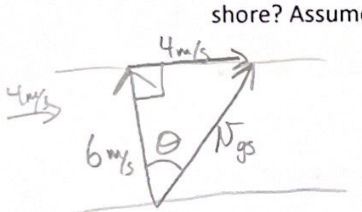
4. Adding velocities $u = \frac{v + u'}{1 + \frac{vu'}{c^2}}$

A spaceship is moving at 0.80c away from Earth. A probe is launched from the spaceship at 0.50c relative to the spaceship, away from Earth. What is the speed of the probe as measured by an observer on Earth?

$$u = \frac{0.8c + 0.5c}{1 + \frac{(0.8\cancel{c})(0.5\cancel{c})}{\cancel{c^2}}} = \frac{1.3c}{1.4} = \underline{\underline{0.93c}}$$

5. Applications of relativity

a) River crossing problem: A river flows due east at 4.0 m/s. A boat is traveling across the river due north at 6.0 m/s relative to the water. What is the boat's speed and direction relative to the shore? Assume the width of the river is 100.m.

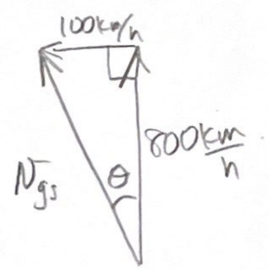


$$v_{gs}^2 = 4^2 + 6^2$$
$$v_{gs} = 7.2\,\frac{m}{s}$$

$$\tan\theta = \frac{4}{6}$$
$$\theta = 34°$$

$$\underline{\underline{\vec{v}_{gs} = 7.2\,\frac{m}{s}\,[34° \text{ E of N}]}}$$

b) Airplane navigation problem: An airplane is flying due north at an airspeed of 800.km/h. There is a wind blowing from the east at 100.km/h. What is the airplane's ground speed and direction? Assume the airplane is flying at an altitude where the wind speed is constant.



$$v_{gs} = \sqrt{100^2 + 800^2} = 806\,\frac{km}{h}$$

$$\tan\theta = \frac{100}{800}$$
$$\theta = 7.13°$$

$$\vec{v}_{gs} = 806\,\frac{km}{h}\,[7.13° \text{ W of N}]$$

Answers:

1a) t_o = 0.60 years, 1b) t = 1.3 s

2a) L = 44 m, 2b) L_o = 5.8 m

3a) m = 2.3 x 10^{-27} kg, 3b) v = 0.866c

4) u = 0.93c

5a) v_{gs} = 7.2 m/s [34° E of N], 5b) v_{gs} = 806 km/h [7.13° W of N]