8.1 Comparing & Interpreting Rates - Key

Comparing and Interpreting Rates [8.1]

Warm up:

Compare different pay scales. Decide if it is better to receive $300 a week or to be paid hourly at a rate of $7.50 per hour.

What factors could affect your decision?

- How many hours a week?

$300 ÷ 7.50 = 40 hours

- What if you work fast?
- What if it takes you more than 40 h. to complete?

Help me decide:

different measures

Orange juice is sold in 1.5L cartons and 250ml boxes. A 1.5L caton sells for $3.75, and ten 250ml boxes sell for $7.39.

Which option is a better deal? (ie. Which size costs less per millilitre?)

Need to Know: 1000 ml = 1 L

1.5 L (×1000) = 1500 ml → $3.75

OR

10 × 250ml = 2500 ml → $7.39

> First thoughts?

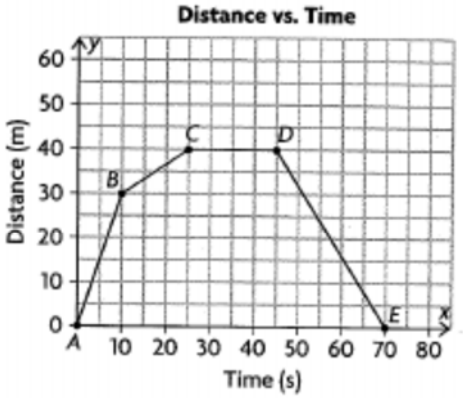
To find the per unit cost:

$\frac{\text{Cost}}{\text{quanity}} \rightarrow \frac{\$3.75}{1500\text{ ml}} = \$0.0025/\text{ml}$ OR $\frac{\$7.39}{2500\text{ ml}} = 0.00295/\text{ml} \rightarrow 0.0030/\text{ml}$

Better Buy

Tell me a story about this:

ex. Oliver left the house and started biking downhill. Then there was a short incline. At the top, he stopped & had some water for 20 minutes.



Then he turned on the turbo booster on his trike and got home really fast.

What do you remember about slope?

Slope = $\frac{y_2 - y_1}{x_2 - x_1}$

* rate of increase / steepness / decline
- change of rates
- "gas price patterns"

Looking back at the graph:

- compare the rates that correspond to each line segment → needs endpoints $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$
- With a partner, discuss why the rates may have changed

Points: A (0,0) B (10,30) C (25,40) D (45,40) E (70,0)
x y

$\text{Slope}_{AB} = \frac{30-0}{10-0}$

$\frac{30}{10}$

$= 3$ m/s

$\text{Slope}_{BC} = \frac{40-30}{25-10}$

$= \frac{10}{15}$

$= \frac{2}{3}$ m/s

OR 0.67 m/s

$\text{Slope}_{CD} = \frac{40-40}{45-25}$

$\frac{0}{20}$

0 m/s

$\text{Slope}_{DE} = \frac{0-40}{70-45}$

$= \frac{-40}{25}$

$= -1.6$ m/s (negative slope)

What do you think happened?

8.2 Solving Problems that Involve Rates - Key

Solving Problems that Involve Rates [8.2]

Getting started with a partner: Which leg of his trip was his fuel efficiency best?

The gas tank of Mario's new car has a capacity of 55L. The owner's manual claims that the fuel efficiency of Mario's car is 7.6L/100km on the highway. Before Mario's first big highway trip, he set his trip to 0km so he could keep track of the total distance he drove. He started with the gas tank full. Each time he stopped to fill up the tank, he recorded the distance he had driven and the amount of gas purchased.

Fill-up	Total Distance Drive (km)	Quantity of Gas Purchased (L)
1	645	48.0
2	1037	32.1

Total distance
So "leg 2" 1037 - 645 = 392 Km

① $\frac{\text{distance}}{\text{fuel}} \rightarrow \frac{645\ \text{Km}}{48\ \text{L}} = 13.44\ \text{Km/L}$ * more fuel efficient*

② $\frac{392\ \text{Km}}{32.1\ \text{L}} = 12.21\ \text{Km/L}$

Trust me… this will really happen to you:

[Insert your name here ☺] wants to defrost a frozen turkey in the microwave. The turkey has a mass of 4.23 kg. A website claims it takes 21 minutes to defrost 3 lbs of meat. ** 1 kg = 2.2 lbs **

How long, to the nearest minute, should you set the timer on defrost for?

↳ How many pounds of Turkey do I have?

$4.23\ kg \times 2.2\ lb = 9.306\ lbs$

- If every 3 lbs needs 21 minutes...

$$\frac{9.306}{3} = 3.102 \times 21\ min = 65\ minutes$$

What we really did ↓

$$4.23\ \cancel{kg} \times \frac{2.2\ \cancel{lb}}{1\ \cancel{kg}} \times \frac{21\ min}{3\ \cancel{lb}} = \frac{196.3143}{3} = 65\ min$$

Bob burns 620 calories in a cardio kick box class lasting 2 h and 120 calories in a body sculpt class lasting 30 min. If he does cardio kick box for 3 hours, how much longer would he have to do body sculpt to burn the same number of calories?

① $\frac{620\ cal}{2\ h} = 310\ cal/h$

② $\frac{120\ cal}{.5\ h} = 240\ cal/h$

- 3 hours of Kick = 3 hours ↙ $3(310) = 930\ cal$

- So, $\frac{930}{240} = 3.875\ hrs.$

Body sculpt per hour →

↳ How much longer → Sculpt time - Kickbox time

$3.875 - 3 = 0.875$ hours OR $0.875\ h \times \frac{60\ min}{} = 52.5\ min.$

Checkpoint - Key

1. Mrs. Hall is training to run a half-marathon, which is about 21.1 km. She can run this distance in 2.2 h. What is her speed in kilometers per hour? Answer to the nearest tenth.

$\frac{\text{distance}}{\text{time}} \rightarrow \frac{21.1\ \text{Km}}{2.2\ \text{h}} = 9.6\ \text{Km/h}$

2.

The butcher shop sells a 4 lb package of chicken legs for $12.57. The supermarket sells chicken legs for $8.68/kg. Which store has the lower price per kilogram? Justify your answer briefly.

TIP
1 kg ≈ 2.2 lb

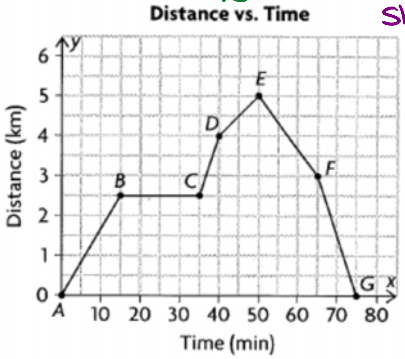
$\frac{4\ \text{lbs}}{2.2} = 1.8\ \text{Kg}$ soo...

B → $\frac{\$12.57}{1.8\ \text{Kg}} = \$6.98/\text{Kg}$ * Butcher sells for less

S → $\$8.68/\text{Kg} = \$8.68/\text{Kg}$

3.

This graph shows how a cyclist travels over time.



a) Over which interval is the cyclist travelling the slowest?
BC → slope = 0, no movement

b) Does the cyclist travel at the same speed over any two intervals? If so, which two?
CD and FG

c) At what speed, in kilometres per hour, is the cyclist travelling in interval *AB*?
10 Km/h

d) Is the cyclist travelling the fastest in interval *EF* or *FG*? Explain.
FG

$\text{Slope}_{DE} = \frac{1}{10} \rightarrow 0.10$

$\text{Slope}_{EF} = \frac{-2}{15} \rightarrow -0.134$

$\text{Slope}_{FG} = \frac{-3}{10} \rightarrow -0.3$

$\text{Slope}_{AB} = \frac{2.5}{15} \rightarrow 0.167$

$\text{Slope}_{BC} = \frac{0}{20} \rightarrow 0$

$\text{Slope}_{CD} = \frac{1.5}{5} \rightarrow 0.3$

4.

It takes 3 h 26 min to fill a 3200 L water tank. Determine the time, in hours and minutes, it will take to fill a 2600 L tank. Round your answer to the nearest minute.

$\frac{3h}{60min} = 180m + 26 = 206m$

$\frac{3200L}{206m} \rightarrow 15.5\ L/m$

② $\frac{2600L}{15.5\ L/m} = 167.7 min$

$167.7 - 120m = 47.7$

2h

2h 48min

5.

It costs $0.3172/lb to ship freight by barge along the Pacific coast. Determine the cost, in dollars, to ship 5360 kg of building supplies from Vancouver to Stewart, B.C. Show your work.

$5360\ kg \times 2.2\ lbs = 11792\ lbs$

→ 11792 (Total weight in lbs)

× 0.3172

$3740.42

6.

Jason wants to ship 40 lb of tinned goods and 50 lb of supplies along the Pacific coast. It costs $5.60/lb, plus a loading fee of $30, to send freight by air and $0.3172/lb to ship freight by water. How much will Jason save by shipping by water instead of by air? Show your work.

40 lbs
50 lbs
90 lbs total

Air → $90\ lbs \times \$5.60 = \$504 + \$30 = \$534.00$

Water → $90\ lbs \times \$0.3172 = \$28.55$

Savings = Air − Water

$\$534.00 - \$28.55 = \$505.45$

8.3 Scale Diagrams - Key

Scale Diagrams [8.3]

$$scale\ factor = \frac{diagram\ measurement}{actual\ measurement}$$

$$scale\ factor = diagram\ measurement : actual\ measurement$$

bigger # on bottom

When a scale factor is between **0 and 1** the new shape will be a **reduction** of the original shape

When the scale factor is **greater than 1** the new shape will be an **enlargement** of the original shape.

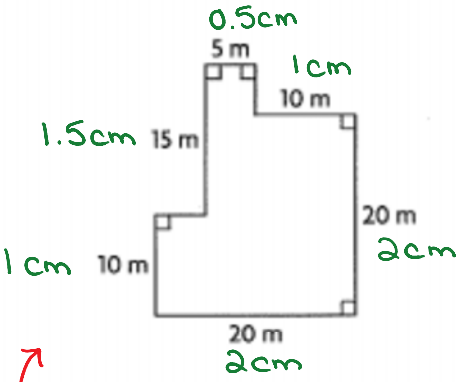
bigger # on top

Example 1:

Create a scale diagram of this building footprint using a scale of

1m : 1000m

$\frac{1}{1000}$ ← Scale factor



We need to calculate the new measurements for every side.

not useful so let's multiply by 100 to get cm.

$5m \times \frac{1}{1000} = 0.005m \times 100 \rightarrow 0.5cm$

$10m \times \frac{1}{1000} = 0.01m \times 100 \rightarrow 1cm$

$15m \times \frac{1}{1000} = 0.015m \times 100 \rightarrow 1.5cm$

$20m \times \frac{1}{1000} = 0.02m \times 100 \rightarrow 2.0cm$

Example 2:

A cross-section of an animal cell is shown ~~is~~ as a scale diagram. In the diagram, the diameter of the cell is 4.5cm. In fact, the cell's diameter is 0.15mm. What scale factor was used to draw the diagram?

to switch to mm

$K = \frac{\text{diagram}}{\text{actual}} = \frac{4.5\text{ cm}}{0.15\text{ mm}} \times 10 \rightarrow \frac{45\text{ mm}}{0.15\text{ mm}} = 300$

Represents unknown scale factor

*not the same unit... still an issue!

$K = \frac{300}{1}$

Scale factor = 300

Example 3:

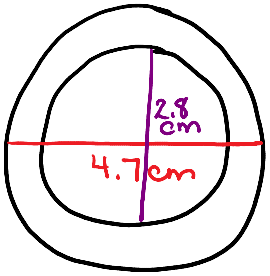
bigger # on top!

BMW wants to enlarge their logo by a scale of $\frac{5}{3}$ so it will fit a new line of clothing. Draw a scale diagram of the logo as it will appear on the clothing.

When we measure the logo we find:

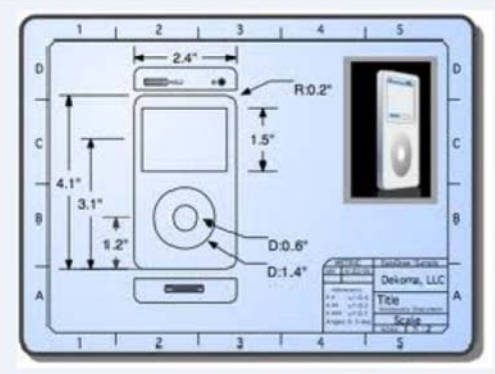
Outside diameter = 2.8cm $\times \frac{5}{3}$ = 4.7 cm

Inside diameter = 1.7cm $\times \frac{5}{3}$ = 2.8 cm



In Class Assignment

What can you do with this information?



8.4 Scale & Area of 2-D Shapes - Key

Scale Factors & Areas of 2-D Shapes [8.4]

$$Area\ of\ similar\ 2D\ shape = k^2(Area\ of\ original\ shape)$$

$$k^2 = \frac{Area\ of\ similar\ 2D\ shape}{Area\ of\ original\ shape}$$

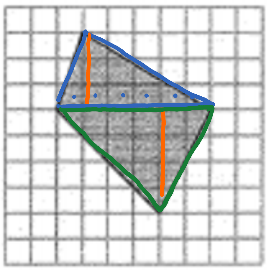
Area = units²

Example 1:

→ No Area formula for a Quadrilateral. We know Area of a Triangle. "Break" shape into 2 Δs.

Determine the area of this quadrilateral, to the nearest tenth of a square unit, after it is reduced by a scale factor of $\frac{1}{4}$.

Area of a Δ = $\frac{Base \cdot Height}{2}$



Δ1 > $\frac{B \cdot H}{2}$

$A_1 = \frac{6 \cdot 3}{2}$

$A_1 = 9\ units^2$

Δ2 > $\frac{B \cdot H}{2}$

$A_2 > \frac{6 \cdot 4}{2}$

$A_2 > 12\ units^2$

So, Area of Quadrilateral = $\Delta A_1 + \Delta A_2$

$9 + 12 = 21\ units^2$ > Now Reduce by $K = \frac{1}{4}$

New Area = $K^2 \cdot$ Original Area

$= \left(\frac{1}{4}\right)^2 (21)$

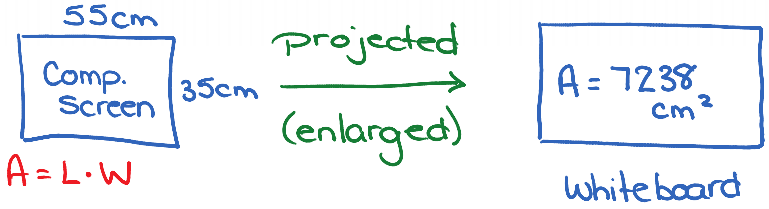
$\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) = \frac{1}{16}$

$= \frac{1}{16} \times \frac{21}{1}$

$= \frac{21}{16}\ units^2$ OR $1.31\ units^2$

Example 2:

A computer screen measures 35cm by 55cm. An image on the computer is projected onto a whiteboard with a screen area of 7238 cm^2. Determine the length and width of the whiteboard.



Area of similar 2D shape = K^2 (Area of original shape)

↳ Area of Whiteboard = K^2 (Area of computer screen)

$7238\ cm^2 = K^2 (55cm \times 35\ cm)$

$\frac{7238\ cm^2}{1925\ cm^2} = \frac{K^2 (1925\ cm^2)}{1925\ cm^2}$ * Want our "K^2" alone

$K^2 = \frac{7238\ cm^2}{1925\ cm^2}$ whiteboard / computer

$K^2 = 3.76$

$K = \sqrt{3.76}$

$K \doteq 1.9$ > Scale Factor

Dimensions of Whiteboard

$35cm \times 1.9 = 66.5\ cm$

$55cm \times 1.9 = 104.5\ cm$

> But when we check...

$66.5 \times 104.5 = 6949.25\ cm^2$

Not $7238\ cm^2$

* This is because of rounding.

If we are exact with our decimals:

$67.9 \times 106.6 = 7238.18\ cm^2$

8.5 Similar Objects - Key

Similar Objects: Scale Models & Scale Diagrams [8.5]

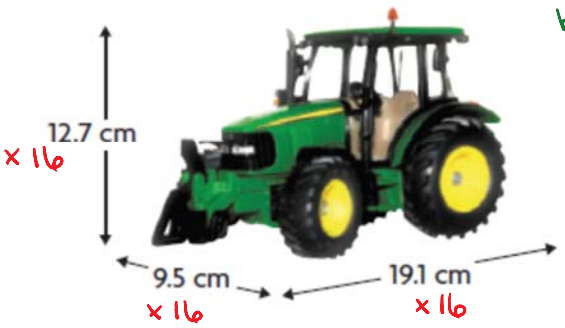
$$k = \frac{Linear\ measurement\ of\ a\ scale\ model}{Corresponding\ linear\ measurement\ of\ object}$$

Still... diagram / actual

Example 1:

Esmeralda bought this toy tractor to give to her younger brother for his birthday. The dimensions of the toy are given in the diagram. The scale ratio on the package is 1:16. She knows that her brother will want to know the size of the real tractor. How can she determine the dimensions of the real tractor?

↳ That's right... a purpose for math after highschool ☺



$K = \frac{1}{16} = \frac{diagram}{actual}$

→ Still scale Factor

You can do this without me. Check if you're right.

→ Is this our best unit?

12.7 x 16 = 203.2 cm ÷ 100 → 2.032 m

9.5 x 16 = 152 cm ÷ 100 → 1.52 m

19.1 x 16 = 305.6 cm ÷ 100 → 3.056 m

Dimensions of Real Tractor

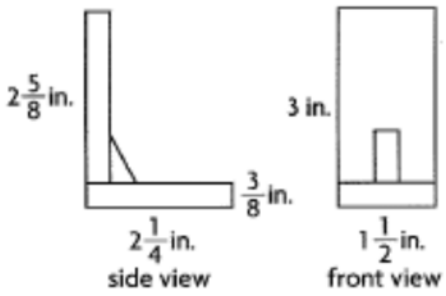
2m x 1.5m x 3m

Example 2:

The plans for a bookend with the scale ratio 1:6 are shown. Determine the dimensions (length, width and height) of the actual bookend.

Scale Factor : $\frac{1}{6}$ so mult. by 6

Prepare yourself for some Fractions:



→ $2\frac{5}{8} \times 6$

$\frac{21}{8} \times \frac{6}{1} = \frac{126}{8} = 15\frac{6}{8}$ Reduces → $15\frac{3}{4}$"

Improper fraction

→ $2\frac{1}{4} \times 6$

$\frac{9}{4} \times \frac{6}{1} = \frac{54}{4} = 13\frac{2}{4} \longrightarrow 13\frac{1}{2}$"

* Best answer (ie. mult. Choice) always in reduced form*

→ $\frac{3}{8} \times \frac{6}{1} : \frac{18}{8} = 2\frac{2}{8} \longrightarrow 2\frac{1}{4}$"

→ $3 \times 6 : 18$"

→ $1\frac{1}{2} \times 6$

$\frac{3}{2} \times \frac{6}{1} = \frac{18}{2} = 9$"

Example 3: No picture? Draw one!

A sewer pipe has an inner diameter of 1.2m, a wall thickness of 0.1m, and length of 3.0m. What measurements should be used to create a scale diagram of the pipe?

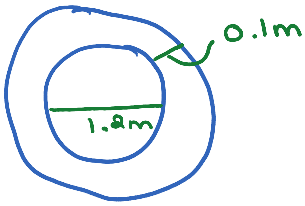
↳ You decide. Why make your choice? There can be alot of different answers. Try to make your life easier.

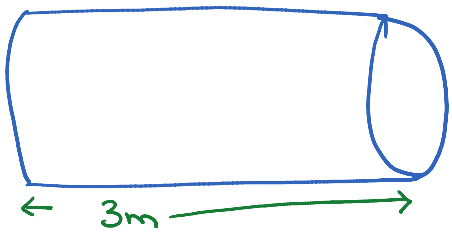
↓

Scale drawing in cm.

↓

Im thinking 12cm for longest piece





② Once I have K:

$1.2\,m \times 0.04 = 0.048\,m$

$0.048m \times 100 = 4.8cm$

$0.1m \times 0.04 = 0.004\,m$

$0.004m \times 100 = 0.4cm$

$3m \times 0.04 = 0.12\,m$

$0.12m \times 100 = 12cm$

①

$3m \times x = 12cm$

Soo...

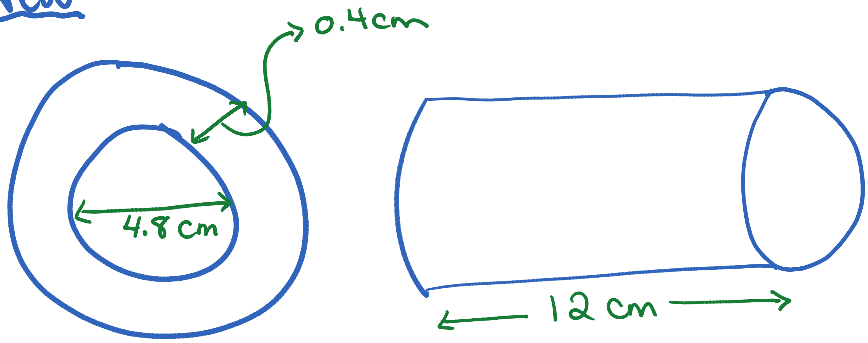
$3m \times x = 0.12m$

$K = \frac{\text{scale model}}{\text{actual}}$

$K = \frac{0.12m}{3m}$

$K = 0.04$ → This is my choice, yours can be different

New



8.6 Scale Factors & 3D Objects - Key

Scale Factors & 3-D Objects [8.6]

Covering what you see with paper

$$surface\ area\ of\ similar\ object = k^2(surface\ area\ of\ original\ object)$$

$$volume\ of\ similar\ object = k^3(volume\ of\ original\ object)$$

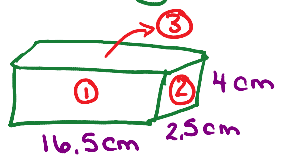
"Filling it up"

Example 1:

$1cm^2 = 0.0001m^2$

A Victorian train engine called "Vicky" was brought from England to Canada to "star" in the play *The Railway Children*. The engine was built in the 1890s and is about 16.5m long. Toy replicas of Vicky are sold at the play. The replicas fit into boxes about 16.5cm long, 2.5 cm wide, and 4.0cm high. Determine the surface area of a shipping container that could hold Vicky.

Shipping Box



Surface Area

① $16.5 \times 4 = 66\ cm^2$

② $2.5 \times 4 = 10\ cm^2$

③ $16.5 \times 2.5 = 41.25\ cm^2$

→ add them → mult by 2 for sides we don't see → $117.25\ cm^2 \times 2 = 234.5\ cm^2$ Surface Area of Similar

Scale Factor:

$$K = \frac{model}{actual} = \frac{16.5\ cm}{16.5\ m \times 100} = \frac{16.5\ cm}{1650\ cm}$$

$$\Rightarrow 0.01 = \frac{1}{0.01}$$

Now: $SA\ Similar = K^2(SA\ original)$

$$\frac{234.5}{(0.01)^2} = \frac{(0.01)^2(x)}{(0.01)^2}$$

$x = 2\,345\,000\ cm^2$

$x = 234.5\ m^2$ *

Example 2:

Determine the volume of a shipping container that could hold Vicky.

$L \times w \times H = units^3$ $\quad 1cm^3 = 0.000001m^3$

Volume of Similar

$16.5 \times 2.5 \times 4$

$= 165\ cm^3$

$V_S = K^3(V_O)$

$$\frac{165}{(0.01)^3} = \frac{(0.01)^3(V_O)}{(0.01)^3}$$

K is still: 0.01

$$V_{orig} = \frac{165}{(0.01)^3}$$

$V_{orig} = 165\,000\,000\ cm^3 \times 0.000001 = 165\ m^3$

Example 3:

Volume

The smaller tank in the photograph has a capacity of 1400 m^3, and the larger tank has a capacity of 4725 m^3.

Spherical tanks are often used to store oil and gas at refineries, since this shape is the most economical to build.

a.) During the refining process, both tanks are filled with oil from a pumping station at the same rate. How many times longer will it take to fill the larger tank than it will take to fill the smaller tank?

b.) How many times greater is the radius of the larger tank than the radius of the smaller tank?

a) $\frac{V_L}{V_S} = \frac{4725}{1400} = \boxed{3.375 = K^3}$

A little more than 3 times as long to fill the tank.

b) $K^3 = 3.375$

$\sqrt[3]{K^3} = \sqrt[3]{3.375}$

$K = 1.5$

The radius of the large tank is 1.5 times larger than the smaller tank.

Ch 8 Review Activity - Key

1.

The surface area of an enlarged triangular prism is 6.25 greater than that of the original prism. How many times greater is the volume of the enlarged prism than the volume of the original?

2.

An orange has a diameter of 7 cm. A grapefruit has a diameter of 12 cm. How many times greater is the volume of a grapefruit than that of an orange?

3.

. The world's tallest building is the Burj Khalifa in Dubai, at 828 m tall. The Calgary Tower in Calgary is 191 m tall. Jermain wants to compare the Burj Khalifa and the Calgary Tower. He has decided to represent the Burj Khalifa with a rectangular prism that has a height of 50 cm. To the nearest tenth of a centimetre, the height of the rectangular prism Jermain should use to represent the Calgary Tower is _____ cm.

4.

The dosage of an antibiotic medicine for a person with a mass of 65 kg is 12 mL. Which equation determines the amount of medicine, P, in millilitres, needed for a person with a mass of 40 kg?

A. $\frac{P}{65\text{ kg}} = \frac{12\text{ mL}}{40\text{ kg}}$

B. $\frac{P}{12\text{ mL}} = \frac{65\text{ kg}}{40\text{ kg}}$

C. $\frac{P}{40\text{ kg}} = \frac{12\text{ mL}}{65\text{ kg}}$

D. $\frac{P}{40\text{ kg}} = \frac{65\text{ kg}}{12\text{ mL}}$

5.

If a bat could fly at its top speed for 2.5 h, it would fly 62.5 km. If an elephant could run at its top speed for 15 min, it would run 10.0 km. Which animal is faster?

A. A bat can fly at 25 km/h and an elephant can run at 40 km/h. The elephant is faster.

B. A bat can fly at 40 km/h and an elephant can run at 25 km/h. The elephant is faster.

C. A bat can fly at 25 km/h and an elephant can run at 40 km/h. The bat is faster.

D. Both animals travel at the same speed.

6.

It takes 3 h 20 min to fill a 5100 L water tank. Determine the time, to the nearest minute, it will take to fill a 6200 L tank.

7.

A 1 : 30 scale model of a tractor is 0.3 ft tall, 0.2 ft wide, and 0.5 ft long. What are the dimensions of the actual tractor?

8.

Two similar objects have a scale factor of 1 : 2.5 associated with them. The volume of the smaller object is 400 cm^3. What is the larger object's volume?

A. 1000 cm^3 **B.** 2500 cm^3 **C.** 6250 cm^3 **D.** none of these

9.

Cylinders A and B are similar. By what factor is the surface area of cylinder B greater than the surface area of cylinder A?