7.1 Exploring Quadratic Relations - Key

Exploring Quadratic Relations [7.1]

Quadratic Equation

A **quadratic equation** is an equation which can be written in the form

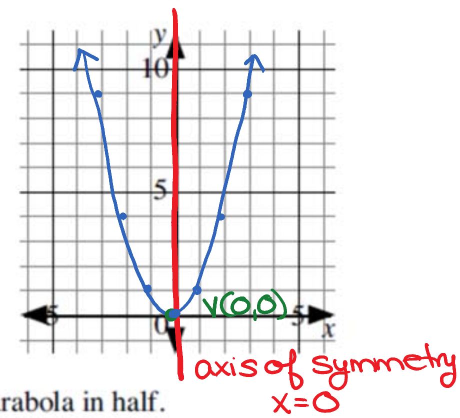
degree = 2 always! that's why $a \neq 0$

$$ax^2 + bx + c = 0, \text{ where } a, b, c \in R, \text{ and } \underline{a \neq 0}.$$

y-int (x=0)

Analyzing the Graph of the Function with Equation $y = x^2$

- Graph the function with equation $y = x^2$ by completing the table of values. Join the points with a smooth curve. The graph of this function is called a parabola.



x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

$(-3)^2$ $(-2)^2$ $(-1)^2$ $(0)^2$ $(1)^2$ $(2)^2$ (3)

(-3,9) (-2,4) (-1,1) (0,0) (1,1) (2,4) (3,9)

- The axis of symmetry is the "mirror" line which splits the parabola in half. State the equation of the axis of symmetry for this parabola.

  $x = 0$

- The vertex of a parabola is where the axis of symmetry intersects the parabola. The vertex can represent a minimum point or maximum point depending on whether the parabola opens up or down.

  Label the vertex (V) on the graph and state its coordinates.

  V(0,0) → "Turning point"

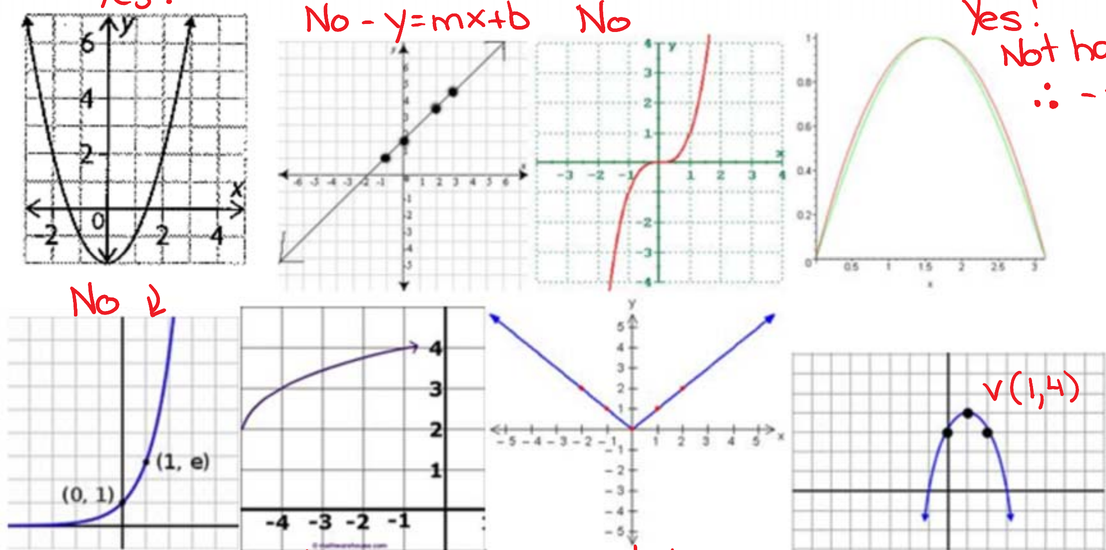
Some Key Ideas

- See x^2 → think ∪ or ∩ "Parabola"
- If "a" is positive → graph is happy and opens up. ∪
  If "a" is negative → graph is sad and opens down ∩

or $y=x^2$

Example 1: Which of the following are **Quadratic relations**?

Yes! No - $y=mx+b$ No Yes! Not happy $\therefore -x^2$



No

Nope! No: $y=|x|$ Yes! $v(1,4)$ $(-x^2)$

- x^2 is highest degree
- no $\sqrt{x}$
- no $\frac{1}{x}$

Example 2: Which of the following are quadratic relations?

Y a.) $y = 3x^2 + 7x \boxed{-2}$ $x=0$ y-int= $(0,-2)$

N b.) $y = x^2 + \sqrt{x}$

Y c.) $y = \boxed{25} - 9x^2$ y-int: $(0,25)$

Y d.) $y = \boxed{7} - 5x^2$ y-int= $(0,7)$

Y e.) $y = 2x^2 \boxed{+11} - 4x$ y-int= $(0,11)$

$y=2x^2-4x+11$

N f.) $y = \frac{1}{4x^2-9x+12}$

N g.) $y = 2x^3 + 6x - 1$

Y h.) $y = (x+2)^2 - 7$

$y=(x+2)^2-7$
$y=(0+2)^2-7$
$y=4-7$
$\boxed{y=-3}$

N i.) $y = 2x - 8$ $y=mx+b$ (line)

Part 2: Determine the y-intercept for each quadratic relation above. $\boxed{}$

$x=0$

Example 3: Does the parabola open up [+a] or down [- a]?

a.) $y = -(2x+5)^2 - 8$

b.) $y = 7 + 2x^2$

c.) $y = -7 + 12x + 3x^2$

d.) $y = -\frac{1}{2}x^2 - 2x + 7$

7.2 Properties of Graphs of Quadratic Functions - Key

Properties of Graphs of Quadratic Functions [7.2]

Warm Up:

This table of values lists points in a quadratic relation.

a) What is the *y*-intercept of the parabola? x=0 (0, -2)

Is this the highest or lowest point on the parabola? (-1, -3) vertex

b) Without graphing, predict the direction in which the parabola opens. Explain how you know.

opens up → from vertex, y values are increasing

x	*y*
−3	1
−2	−2
−1	−3
0	−2
1	1
2	6

x=0 →

→ Vertex

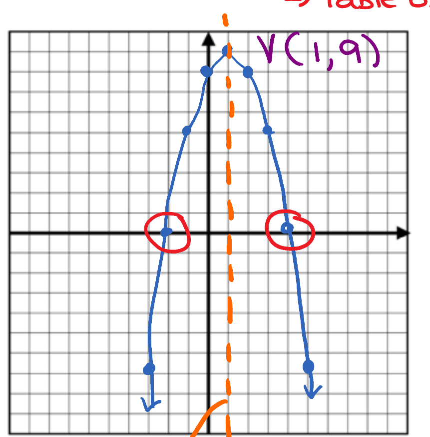
Example 1:

Determine the y-intercept (x=0), any x-intercepts (y=0), the equation of the axis of symmetry, the coordinates of the vertex, and the domain and range of the function

$f(x) = -x^2 + 2x + 8$. Sketch the graph.

Domain: Dx
Range: Ry

↳ Table of Values



x-int →

axis of symmetry
x = 1

X	y
-3	-7
-2	0
-1	5
0	8
1	9
2	8
3	5
4	0

I always start with these #s.

$y = -x^2 + 2x + 8$ → once you understand your calculator will do this.

$y = -(-3)^2 + 2(-3) + 8$

$y = -(9) - 6 + 8$

$y = -9 - 6 + 8$

$y = -7$

→ y int > x=0

$y = -(0)^2 + 2(0) + 8$

$\boxed{y = 8}$

x-int > y=0

$\boxed{(-2,0);(4,0)}$

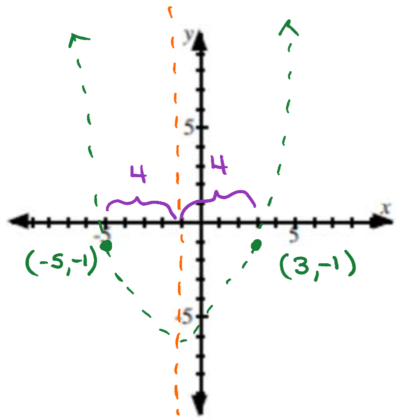
Domain: $x \in \mathbb{R}$

Range: $y \leq 9$

Example 2:

$x = ?$ (x, y) (x, y)

Determine the equation of the axis of symmetry if $(-5, -1)$ *and* $(3, -1)$ are located on the parabola.



Notice they have the same y value → use midpoint formula

$$\frac{x_1 + x_2}{2}$$

$$\frac{-5+3}{2}$$

$$\frac{-2}{2}$$

$$= -1$$

$$\boxed{\therefore x = -1}$$

Example 3:

blah, blah, blah...

Some children are playing at the local splash pad. The water jets spray water from ground level. The path of water from one of those jets forms an arch that can be defined by the function

h=0

opens down Quadratic!

$$\underset{y}{f(x)} = -0.12x^2 + 3x + 0$$

→ y-int at (0,0)

where x represents the horizontal distance from the opening in the ground in feet and $f(x)$ is the height of the sprayed water, also measured in feet. What is the maximum height of the arch of water, and how far from the opening in the ground can the water reach? y=0

No negatives
Real Place,
Whole #s

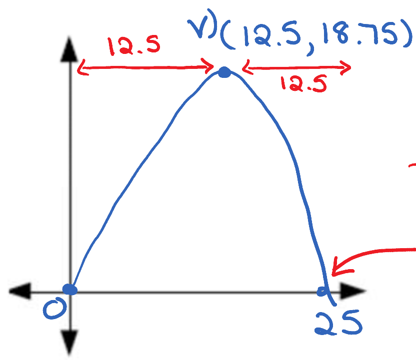
Use calculator! $y = -0.12(2)^2 + 3(2)$

x	y
0	0
2	5.5
4	10.08
6	13.62
8	16.32
10	18
12	18.72
13	18.72
14	18.48

same! Vertex must be in the middle

* starts going back down

V(12.5, 18.75)



$$\begin{array}{r} 12.5 \\ +12.5 \\ \hline 25 \end{array}$$

max height = 18.75 ft.
Water reaches 25 ft.

7.3 Solving By Graphing - Key

Solving Quadratic Equations by Graphing [7.3]

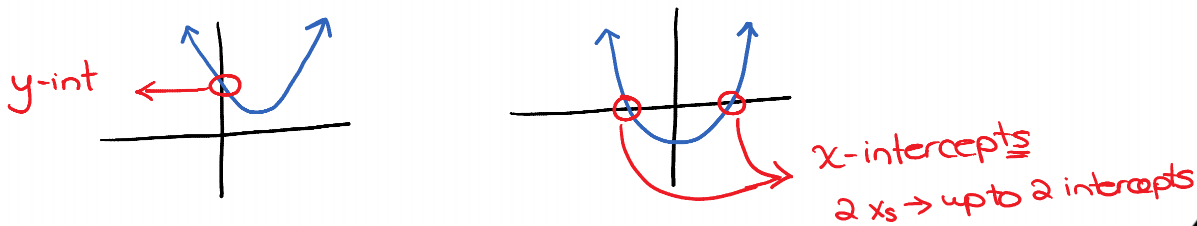
Complete my thoughts…

- Y-intercept means $x = 0$ and that is where the graph crosses the y axis.

Sooo….

- X- intercept must be when $y = 0$ and that is where the graph crosses the x axis.

Draw me a sketch:



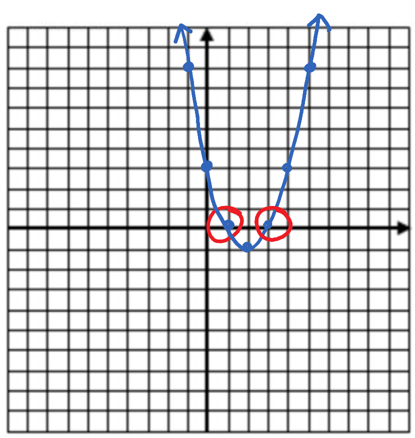
Solve → Find x= / find x-int(s)

Example 1:

Solve $x^2 - 4x + 3 = 0$ by graphing and determine the zeros.

→ (x,0) ; (x,0)

→ Table of Values



x	y
-3	24
-2	15
-1	8
0	3
1	0
2	-1
3	0

> don't use. #s aren't useful

→ Vertex

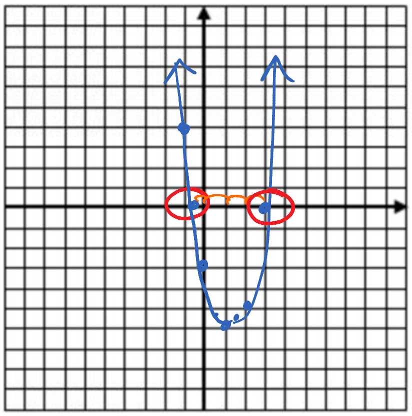
Solutions > $\boxed{x = 1, 3}$

Example 2:

Solve $2x^2 - 5x - 3 = 0$ by graphing and determine the zeros.

x =

→ Table of Values



x	y
-3	30
-2	15
-1	4
0	-3
1	-6
2	-5
3	0
-0.5	0

> Big jump?

No pattern yet

✓ aha!

Find Vertex

① $\frac{-0.5+3}{2} = \frac{2.5}{2} = 1.25$

V(1.25, -6.125)

② $2(1.25)^2 - 5(1.25) - 3$
$= -6.125$

$\boxed{x = -0.5, 3}$

Example 3:

Solution will be true to both.

Solve $3v(v+3) = 2(v^2-4)$ by graphing the expressions on both sides of the equation.

① $3v(v+3)$
$3v^2 + 9v$

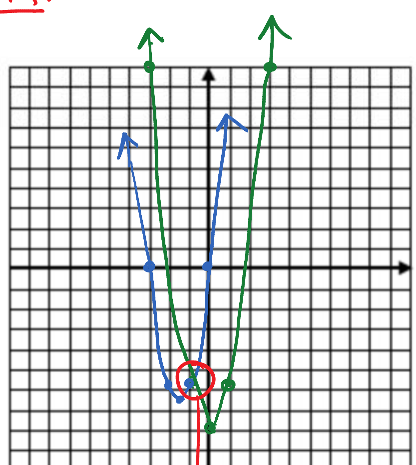
x	y
-3	0
-2	-6
* -1	-6
0	0
1	
2	
3	

* V(-1.5, -6.75)

② $2(v^2-4)$
$2v^2 - 8$

x	y
-3	10
-2	
-1	-6
0	-8
1	-6
2	
3	10

→ Vertex and y-int



1 Solution.
The other solution is off our graph.

7.4 Factored Form - Key

Factored Form of a Quadratic Function [7.4]

QUADRATIC	INTERCEPT	FACTOR
• ax^2+bx+c • $a \neq 0$	• where the graph crosses an axis y-int: $x=0$ crosses y axis x-int: $y=0$ crosses x axis	

Let's review some factoring:

List the factors of 20:

$20 \cdot 1$
$4 \cdot 5$
$2 \cdot 10$
$-2 \cdot -10$
$-4 \cdot -5$
$-20 \cdot -1$
$-2 \cdot -2 \cdot 5$

The GCF = Greatest Common Factor

Try these:

$5x^2 + 10x$

$5x(x+2)$

$3x^2 + 3x - \cancel{2}36$

$3(x^2 + x - 12)$

$3(\quad)(\quad)$ → anyone?

$3(x+4)(x-3)$

Factoring Trinomials – What do you remember?

• Take out a GCF 1st
• factors into 2 brackets ( )( )

Try these:

An easier one > $x^2 + x - 6$

$x \cdot x$ $\quad$ $6 \cdot 1$, $3 \cdot 2$

$(x+3)(x-2)$

$+3x$

$-2x$

x

Harder one > $2x^2 + 3x - 2$

$2x \cdot x$ $\quad$ $2 \cdot 1$

$(2x - 1)(x + 2)$

$-1x$

$+4x$

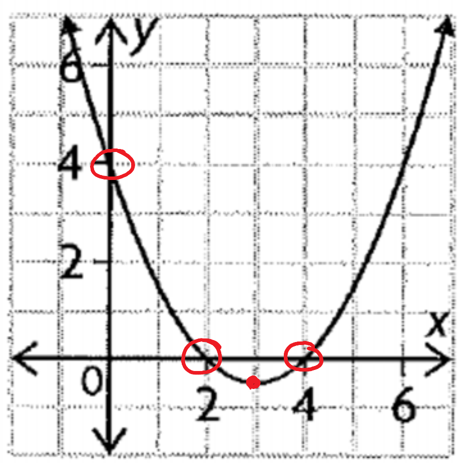
$3x$

*Placement is going to matter.

Looking at this graph, what characteristics do you think are important & why?

(Feel free to write / sketch/ indicate/ etc to get your point across)

y-int (x=0)



x-ints (y=0)

V(3,-0.5)

↳ Turning Point
- Axis of Sym
- max or min
- Range

When a quadratic function is written in factored form

$$Y = a(x - r)(x - s)$$

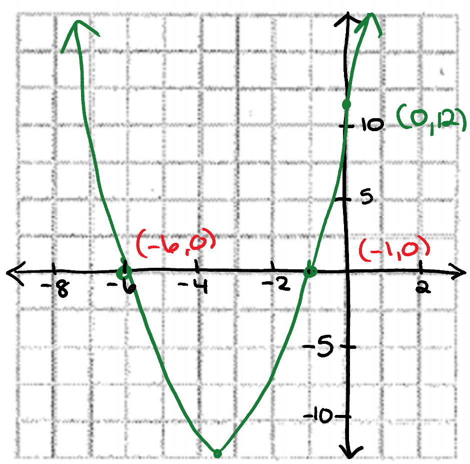
Example 1:

Sketch the graph of the quadratic function: $f(x) = 2x^2 + 14x + 12.$

positive x^2

y-int (0,12)

No table of values



① Factor - check for GCF

$2(x^2+7x+6)$ $\begin{matrix}-6\cdot 1\\-3\cdot 2\end{matrix}$

② Factor into 2 brackets

$y = 2(x+6)(x+1)$

③ To find x-int (zeros) look at each factor separtely.

$x+6 = y$ | $x+1 = y$
$x+6 = 0$ | $x+1 = 0$
$\underline{\underline{x = -6}}$ | $\underline{\underline{x = -1}}$

x-int.

④ y-int → $x = 0$

$y = 2(x+6)(x+1)$

$y = 2(0+6)(0+1)$

$y = 2(6)(1)$

$\underline{\underline{y = 12}}$

⑤ Vertex → Find midpoint of x-int.

$$\frac{-6+(-1)}{2} = \frac{-7}{2} = -3.5$$

If $x = -3.5$ y is...

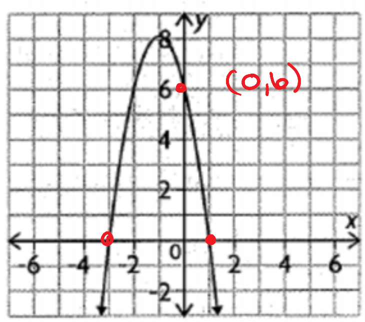
$y = 2(-3.5+6)(-3.5+1)$

$y = 2(2.5)(-2.5) \longrightarrow y = -12.5$

$\boxed{V(-3.5, -12.5)}$

Example 2:

Determine the quadratic function that defines this parabola. Write the equation of the function in standard form.



$x = -3 \quad x = 1$

x-int.

y-int: (0,6)
x, y

x-int

$y = a(x-r)(x-s)$

$y = a(x-(-3))(x-(+1))$

$y = a(x+3)(x-1)$ → still have too many unknowns Use y-int

$6 = a(0+3)(0-1)$

$6 = a(3)(-1)$

$\frac{6}{-3} = \frac{-3a}{-3}$

$\underline{\underline{a = -2}}$

Put all the pieces together: $y = -2(x+3)(x-1)$

To switch form (standard form) use FOIL to expand.

$y = -2(x+3)(x-1)$

$y = -2(x^2 - x + 3x - 3)$

$y = -2(x^2 + 2x - 3)$

$\boxed{y = -2x^2 - 4x + 6}$

7.5 Solving Quadratics by Factoring - Key

Solving Quadratic Equations by Factoring [7.5]

- x-int
- y-int
- vertex
- direction

Let's start slow. What do you need to know to graph: $y = x^2 + 8x + 15$

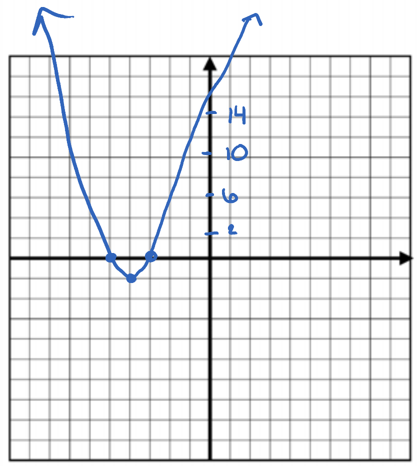
Can you do it?

$x^2 + 8x + 15$

$(x+3)(x+5)$

$x = -3, -5$

y-int



Vertex: ① $\frac{-3+(-5)}{2}$, $\frac{-8}{2} = -4$

② $y = x^2 + 8x + 15$

$y = (-4)^2 + 8(-4) + 15$

$y = -1$

③ $V(-4, -1)$

What patterns do you notice with the following numbers? Perfect squares

	$3\cdot3$		$5\cdot5$		$7\cdot7$		$8\cdot8$		$x\cdot x$		♪ · ♪
4	9	16	25	36	49	64	81	100	x^2	y^2	♪^{2}
$2\cdot2$		$4\cdot4$		$6\cdot6$		$8\cdot8$		$10\cdot10$		$y\cdot y$	

What do you notice about the following quadratic? Perfect square trinomial

$x \cdot x$ $\quad$ $6\cdot6$

$$x^2 + 12x + 36 = 0$$

$(x+6)(x+6)$

Can you show me another way to represent it?

$(x+6)^2$

$\rightarrow x = ?$

Using all the tricks you've re-visited so far. Solve the following equation.

* I know it's tricky… do it anyways!

so it's equal to zero, then factor

$$9x^2 + 42x = -49$$

$$9x^2 + 42x + 49 = 0$$

$$(3x+7)(3x+7)$$

$$3x+7=0$$

$$3x=-7$$

$$x = \frac{-7}{3}, \frac{-7}{3}$$

How do you know your answer is right?

- Replace x with $-\frac{7}{3}$. If right side = left side > you're good
- Factor brackets using FOIL to check.

Making it work in reverse. What quadratic equation could have the roots – 6 and 8?

$ax^2+bx+c=0$ x-int, zeros,

$$x = -6 \qquad x = 8$$

$$x+6=0 \qquad x-8=0$$

$$(x+6)=0 \qquad (x-8)=0$$

$$(x+6)(x-8)=0$$ → Use Foil to expand

$$(x+6)(x-8)=0$$

$$x^2-8x+6x-48=0$$

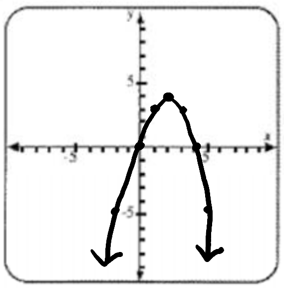
$$x^2-2x-48=0$$

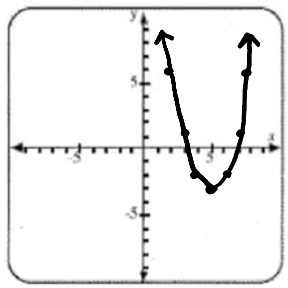
$$\boxed{x^2-2x-48=0 \text{ OR } y=x^2-2x-48}$$

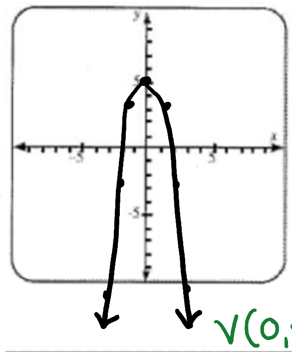
Time to work on the assignment – then a quick write to show what you know

Check In - Key

1 > For each of the graphs below please state:







Vertex:	V(2,4)	V(5,-3)	V(0,5)
Max/Min:	max	min	max
Axis of Sym:	$x=2$	$x=5$	$x=0$
Y -int:	(0,0)	?	(0,5)
Domain:	$x \in R$	$x \in R$	$x \in R$
Range:	$y \leq 4$	$y \geq -3$	$y \leq 5$

x = ?

2 > Solve for x: $-2x^2 + 2x + 40 = 0$ GCF = -2

$-2(x^2+x+20)=0$ Factor

$-2(x+5)(x-4)=0$

$\boxed{x=-5, +4}$

What this tells me:

- Graph opens down
- y-int = (0,40)
- x-int = (-5,0), (4,0)
- I could find the vertex if I wanted to.

3 > Solve this quadratic equation by factoring

$$8(0.125x^2 - 0.875x = -1.5)$$

Ugly! What can I multiply everything by to make it prettier?

$$\frac{1}{0.125} = 8$$

$$8(0.125) = 1$$

$$x^2 - 7x = -12$$

$$+12 \quad +12$$

$$x^2 - 7x + 12 = 0$$

$$(x-3)(x-4) = 0$$

$$\boxed{x = 3, 4}$$

→ graph opens up ↻

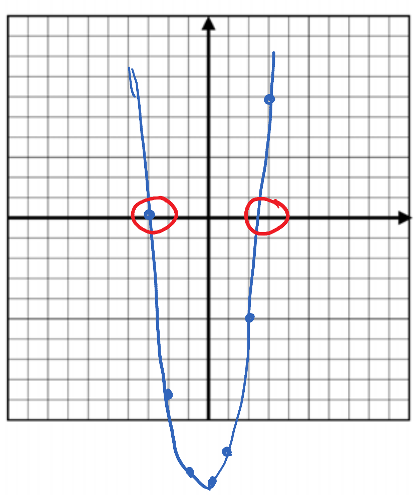
→ x-int: (3,0) (4,0)

→ y-int: (0,12)

x=

Table of values

4 > Solve by graphing: $2y^2 + y - 15 = 0$



x	y
-3	0
-2	-9
-1	-14
0	-15
1	-12
2	-5
3	6

$$\boxed{x = -3, 2.5}$$

7.6 (1) Vertex Form - Key

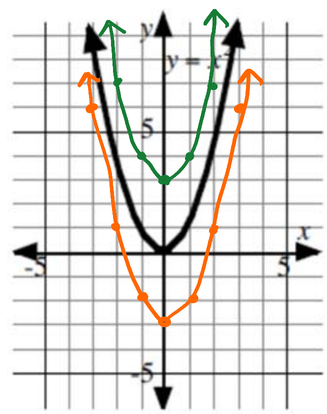
Vertex Form of a Quadratic Function [7.6]

Quadratic functions can be written in **general form** or **vertex form**.

General Form: $y = ax^2 + bx + c$
↳ Needs table of Values

Vertex Form: $y = a(x - p)^2 + q$ → Easy to graph
Standard Form

Investigation #1 Analyzing the Graph of $y = x^2 + q$



The graph of $y = f(x) = x^2$ is shown.

a) Write an equation which represents each of the following:

- $y = f(x) + 3$

$y = 1(x-0)^2 + 3$
$V(0,3)$

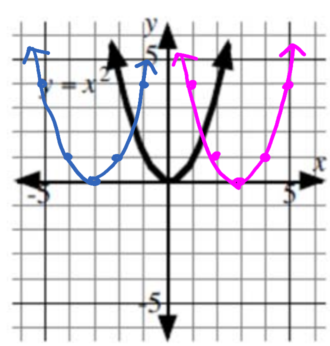
- $y = f(x) - 3$

$y = 1(x-0)^2 - 3$
$V(0,-3)$

b) ~~Use a graphing calculator to~~ sketch $y = f(x) + 3$ and $y = f(x) - 3$ on the grid.

What do we know? q represents the "y" of my vertex. It's sign doesn't change. It moves my vertex "up" or "down".

Investigation #2 Analyzing the Graph of $y = (x - p)^2$



The graph of $y = f(x) = x^2$ is shown. → $y = 1(x-0)^2 + 0$

a) Write an equation which represents each of the following:

- $y = f(x + 3)$

$y = 1(x+3)^2 + 0$
$V(-3,0)$

- $y = f(x - 3)$

$y = 1(x-3)^2 + 0$
$V(3,0)$

b) ~~Use a graphing calculator to~~ sketch $y = f(x + 3)$ and $y = f(x - 3)$ on the grid.

What did we learn? P represents the "x" of my vertex. The sign changes when it comes out of the bracket. It moves my graph left or right.

Putting it all together. How would you explain <u>Vertex Form</u> to someone else?

+a = ↻
- a = ↺

Vertex

$y = a(x-p)^2 + q$ → moves graph up or down

moves graph left or right

* We like this form because its easier to graph!

Show me you understand:

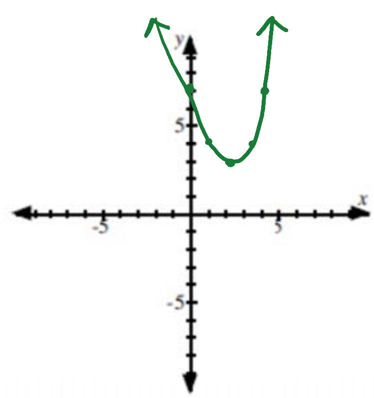
Function	*Equation Representing Function*	*Vertex*	*Max/Min Value*	*Equation of Axis of Symmetry*
$y = f(x)$	$y = x^2$	$(0, 0)$	min, 0	$x = 0$
$y = f(x+2) - 4$	$y = 1(x+2)^2 - 4$	$(-2, -4)$	min, -4	$x = -2$
$y = f(x-p) + q$	$y = 1(x-p)^2 + q$	(p, q)	min, q	$x = p$

Time to show off:

Consider the graph of the function $f(x) = (x-2)^2 + 3$.

a) Without using a graphing calculator, sketch the graph on the grid.

$y = 1(x-2)^2 + 3$



b) State the coordinate of the vertex. $V(2, 3)$

c) State the maximum or minimum value of the function.

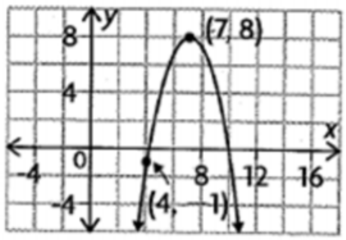
↻, min value at 2

d) State the domain and range of the function.

$D_x: x \in R$

$R_y: y \geq 3$

Example: Determine the quadratic function corresponding to this parabola.



What do we know?

opens down: $-x^2$

vertex: $(7,8)$
(p,q)

Point: $(4,-1)$
(x,y)

What can we use?

$y = a(x-p)^2+q$

Need to solve for a.

$y = a(x-p)^2+q$

$-1 = a(4-7)^2+8$

$-1 - 8 = a(-3)^2+8 - 8$

$\frac{-9}{9} = \frac{9a}{9}$

$\boxed{a=-1}$

So

$\boxed{y=-1(x-7)^2+8}$

7.6 (2) Vertex Form - Key

Vertex Form of a Quadratic Function – Day 2 [7.6]

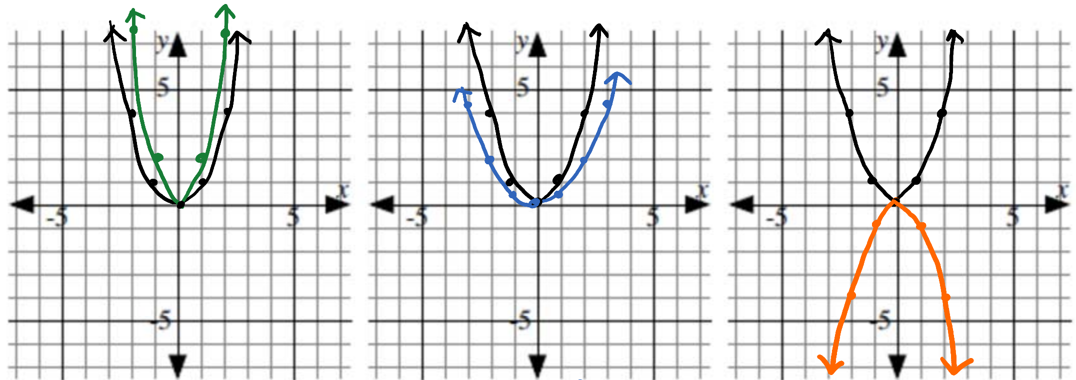
Vertex Form. What does that look like again?

$$y = a(x-p)^2 + q$$

So what about "a" ?

"Tall + Skinny" $a > 1$ | "Short + fat" $0 < a < 1$ | $a < 1$

x^2



$y = 2x^2$ | $y = \frac{1}{2}x^2$ | $y = -x^2$

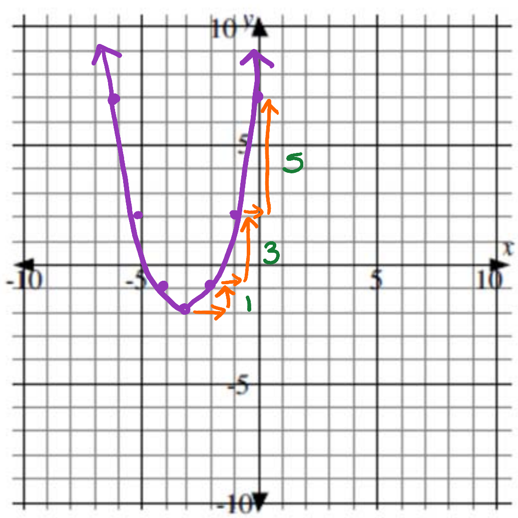
Graphing with the Method of Differences

MOD: 1, 3, 5

Graph: $y = (x+3)^2 - 2$

What do we know?

- $V(-3,-2)$
- $a = 1$
  ↳ ↑↑
  - 1 → over 1, up 1
    3 → over 1, up 3
    5 → over 1, up 5



We also know:

$D_x: x \in R$

$R_y: y \geq -2$

axis of sym: $x = -3$

- minimum value: -2

Sketch the graph of the quadratic function $y = -2(x + 4)^2 + 10$.

State the domain & range of the function.

$V(-4, 10)$

$a = -2$

↳ ∩

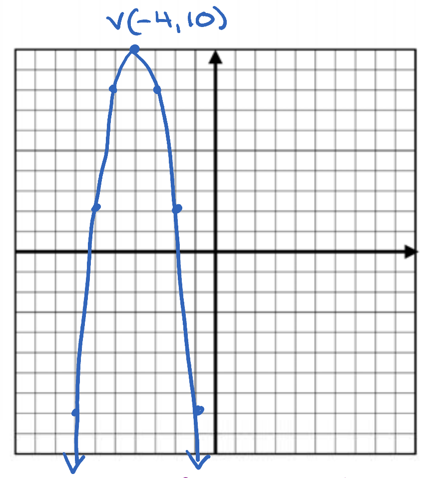
↳ $1 \cdot 2 = 2$ → over 1, down 2

$3 \cdot 2 = 6$ → over 1, down 6

$5 \cdot 2 = 10$ → over 1, down 10

$D_x: x \in R$

$R_y: y \leq 10$



* To find y-int we'd have to let x = 0

Applying it all to Real Life...

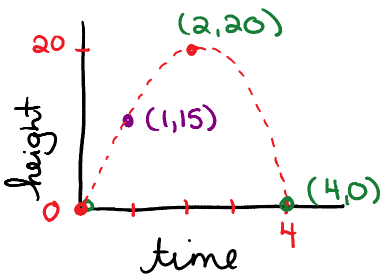
A soccer ball is kicked from the ground ($h=0$). After 2s the ball reaches its maximum height of 20m. It lands on the ground at 4s. Point $(4,0)$ [x, y] $V(2,20)$

a.) Determine the quadratic function that models the height of the kick.

b.) Determine any restrictions that must be placed on the domain and range of the function.

c.) What was the height of the ball at 1s? When was the ball at the same height on the way down?

Think about it...



a) $(p, q) = (2, 20)$
$(x, y) = (4, 0)$ →

$y = a(x-p)^2 + q$

$0 = a(4-2)^2 + 20$ (-20, -20)

$-20 = a(2)^2$

$\frac{-20}{4} = \frac{4a}{4}$

$a = -5$

$$\boxed{y = -5(x-2)^2 + 20}$$

b) Only Quad I - has a start and finish.

$D_x: 0 \leq x \leq 4$

$R_y: 0 \leq y \leq 20$

c) $x = 1 \text{ sec}$ > $y = -5(1-2)^2 + 20$

$y = -5(-1)^2 + 20$

$y = -5 + 20$

$\boxed{y = 15 \text{ m}}$

7.7 Quadratic Formula - Key

Using the Quadratic Formula [7.7]

The Quadratic Formula

The quadratic equation $ax^2 + bx + c = 0, a \neq 0$ has the roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

We love the Quadratic Formula! Factoring... Watch out!

Before we get started, a few helpful hints:

- Use brackets when replacing a letter → don't want to lose a negative
- If you get a negative under the radical = no solution → doesn't cross the x-axis
- Show all your work the 1st time → it's easier than doing it again
- have fun... We ❤ the QF!

* Don't forget Bedmas

Example 1: Solve the quadratic equation $4x^2 - 3 = 7x$. Give an exact answer and an approximate answer to 3 decimal places.

$x =$

$\boxed{a}x^2 + \boxed{b}x + \boxed{c} = 0$

$\boxed{4}x^2 \boxed{-7}x \boxed{-3} = 0$

$a = 4$
$b = -7$
$c = -3$

↑ I write this every time...

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(4)(-3)}}{2(4)}$$

$$x = \frac{+7 \pm \sqrt{49 + 48}}{8}$$

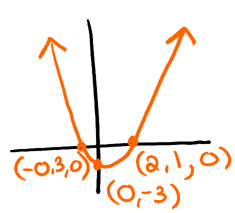
$$\boxed{x = \frac{7 \pm \sqrt{97}}{8}}$$

exact answer

$$x = \frac{7 + \sqrt{97}}{8} \quad \text{and} \quad x = \frac{7 - \sqrt{97}}{8}$$

$\boxed{x = 2.106}$ $\boxed{x = -0.356}$

Model this →



Try this one: Solve the quadratic equation $2x^2 + 8x - 5 = 0$

$a = 2$
$b = 8$
$c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(2)(-5)}}{2(2)}$$

$$x = \frac{-8 \pm \sqrt{64 + 40}}{4}$$

$$x = \frac{-8 \pm \sqrt{104}}{4}$$

big number, can we reduce?

$\sqrt{104} = \sqrt{4 \cdot 26}$
$\sqrt{4} \cdot \sqrt{26}$
$2\sqrt{26}$

$$x = \frac{-\cancel{8}^{4} \pm \cancel{2}^{1}\sqrt{26}}{\cancel{4}\,2}$$

$$\boxed{x = \frac{-4 \pm \sqrt{26}}{2}}$$

$x = \frac{-4 + \sqrt{26}}{2}$ and $x = \frac{-4 - \sqrt{26}}{2}$

$\boxed{x = 0.550}$ $\boxed{x = -4.550}$

x=
Solve the quadratic equation $x^2 + 9x + 23 = 0$. Draw a really quick sketch to demonstrate what you found out.

$a = 1$
$b = 9$
$c = 23$

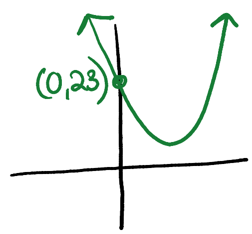
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(9) \pm \sqrt{(9)^2 - 4(1)(23)}}{2(1)}$$

$$x = \frac{-9 \pm \sqrt{81 - 92}}{2}$$

$$x = \frac{-9 \pm \sqrt{-11}}{2}$$

No Real Solutions!
(ie. Stop here ü)



Working with a partner, write the steps to solve this problem.

A store rents an average of 750 video games each month at a current rate of $4.50. The owners of the store want to raise the rental rate to increase the revenue to $7000 per month. However, for every $1 increase, they know that they will rent 30 fewer games each month. The following function relates the price increase, *p*, to the revenue, *r*.

$$(4.5 + p)(750 - 30p) = r$$

Can the owners increase the rental rate enough to generate revenue of $7000 per month?

1. Replace "r" with $7000
2. Use FOIL to expand
3. We want it $= 0$, -7000 from both sides
4. Write it how we like it
5. Use Quadratic Formula to solve for x
   $a = -30$ $b = 615$ $c = -3625$
6. What did we find out?

$$(4.5 + p)(750 - 30p) = 7000$$

$$-7000 \quad 3375 - 135p + 750p - 30p^2 = 7000 \quad -7000$$

$$-3625 + 615p - 30p^2 = 0$$

$$-30p^2 + 615p - 3625 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(615) \pm \sqrt{(615)^2 - 4(-30)(-3625)}}{2(-30)}$$

$$x = \frac{-615 \pm \sqrt{-56775}}{-60}$$

→ No Real Solution

→ It's not possible for the owners to make $7000.

7.7 Worksheet - Key

7.7 Solving Quadratic Equations Using the Quadratic Formula

Keep in Mind

- The roots of a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$, can be determined using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- You can use this formula to determine the roots, if they exist, of any quadratic equation, even if it is not factorable.
- The quadratic formula will give you an exact value for the solution.
- If the radicand, $b^2 - 4ac$, simplifies to a perfect square, then the equation can be solved by factoring.
- If the value of the radicand is negative, then the equation has no real solution.

2. Suppose you were to solve these equations using the quadratic formula. What values of a, b, and c would you use in each case?

a) $3x^2 - 2x + 1 = 0$

b) $-2(x - 1)^2 - 1 = 0$

4. Solve each quadratic equation. Identify any equations that do not have real roots. Otherwise, give an exact answer.

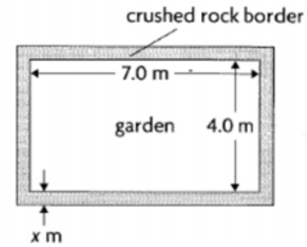
a) $2x^2 - x - 3 = 0$

c) $16x^2 + 8x + 3 = 0$

b) $5x^2 = 6x + 2$

d) $-2x^2 + 8x = 3$

5. A landscaper is designing a rectangular garden, as shown. She has enough crushed rock to cover an area of $10.0\ m^2$ and wants to make a uniform border around the garden. How wide should the border be, if she wants to use all the crushed rock?



6. On Mars, a ball thrown from the top of a spacecraft 6.5 m high could be modelled by $h(t) = -1.89t^2 + 5t + 7.5$. On Neptune, a ball thrown from the top of the same spacecraft could be modelled by $h(t) = -7.0t^2 + 5t + 7.5$. In these equations, h is the height in metres and t is the time in seconds. How much earlier would a ball fall to the base of the spacecraft on Neptune than on Mars? Give your answer to the nearest hundredth of a second.

on Mars: $t =$ _____ (Choose the positive root.)

on Neptune: $t =$ _____

The ball would fall to the base of the spacecraft _____ earlier on Neptune.

7. Suppose a pebble were to fall from a 200 m cliff to the water below. The height of the stone, $h(t)$, in metres, after t seconds can be represented by the function $h(t) = -4.9t^2 + 3t + 200$. How long would the stone take to reach the water, to the nearest tenth of a second? Show your work.

2. a) $a = 3, b = -2, c = 1$ b) $a = -2, b = 4, c = -3$

4. a) $x = 1.5$ or $x = -1$

b) $x = \dfrac{3 - \sqrt{19}}{5}$ or $x = \dfrac{3 + \sqrt{19}}{5}$

c) no real roots

d) $x = \dfrac{4 - \sqrt{10}}{2}$ or $x = \dfrac{4 + \sqrt{10}}{2}$

5. $\doteq 0.4$ m

6. on Mars: $t =$ 3.713...

on Neptune: $t =$ 1.452...

The ball would fall to the base of the spacecraft 2.26 s earlier on Neptune.

7. $\doteq 6.7$ s